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THE MATHEMATICS TEACHER

Volume XLV Number 5

Mathematical Preparation for College*

By

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There are three groups of persons who are continually concerned with the question of who should study what mathematics in high school. These groups are: (1) the students themselves, (2) the high school teachers and administrators who counsel the students and plan and teach their courses, (3) the college teachers and administrators who set up requirements for college courses and then teach the students who enroll in accord with them, and who, incidentally, have the responsibility for the training of prospective teachers.

Most of this article is addressed directly to high school students. Their need for mathematical preparation is discussed with three specific suggestions for all college preparatory students. Requirements of many college curricula are listed, and many curricula are classified according to their mathematical requirements. The remainder of this article is then addressed to teachers and administrators of both the secondary schools and the colleges.

High Schoolers: From the ninth grade on, each of you regularly has to answer for yourself the question, "More mathematics—to take it or not to take it?"[1] To answer this question you need to know the facts about where high school mathematics may be needed. This article gives you one set of these facts; namely, those that should be known to everyone who might go to college.

But first, let's be frank about who is writing this and why it is being written. The people writing this note to you are mathematicians who believe that mathematics is important, useful, fun, and interesting. We know that the lists which we shall give here include only a few of the interests, activities, and studies which real, live human beings inevitably find are all mixed up with mathematics. We know that mathematics is neither dry nor dead. We know that on the contrary, it is today in a period of great expansion and interesting progress. But we are not, here, trying to sell you any of these facts. Mathematics is important in many vocations (see the

* Bound reprints of this article may be obtained from our Washington office for 20¢ each, postpaid.

[1] *A Mathematics Student—To Be or Not to Be?* was the title of a pamphlet published and distributed to Michigan high schools by a committee of the Michigan Section of the Mathematical Association of America composed of Howard Alexander of Adrian College, Phillip S. Jones of the University of Michigan, and Cleon C. Richtmeyer (Chairman) of Central Michigan College of Education.

The first part of the present article is largely based on this pamphlet which is now out of print.

For further discussion of the work of this committee, see Cleon C. Richtmeyer, "A Program of Information for Prospective College Students," *American Mathematical Monthly*, LVI (Feb. 1949), 90–91.

references in footnote 2 if you want more information about this). Mathematics is useful to you as a consumer. Mathematics is like music, in that many who live without appreciating it would live no longer but much more pleasantly if they did find pleasure in some understanding of it. But, though all these statements are true, it is not of these facts either that we wish to talk here.

What, then, *are* we trying to do? We want to help those of you who may go to college to avoid trouble and disappointment. We want to eliminate the possibility that you will be refused permission to take necessary college courses because of inadequate mathematical preparation in high school. We want to help you find and prepare for your careers without any of the delays which often result from inadequate preparation for college.

Don't misunderstand us. *All* American citizens need to know *some* mathematics. We are not, here, discussing that common mathematical knowledge which should be possessed by all, nor the mathematics required for various vocations. We are talking of the needs of students who are going to college. In discussing their needs, we will use the names "algebra" and "geometry" since courses with these names are commonly found in high schools. However, when these courses are listed as required, the important thing is not that you have credit in courses with these names but that you have *mastered* the basic concepts and acquired the important skills which are included in such courses. Such knowledge may be acquired in courses with names other than "algebra" and "geometry." The understanding of ideas is important; the course names are immaterial.

Of course you can get into many colleges without having had algebra and geometry. You can even take many courses in college after being so admitted. However, the Michigan newspaper which recently printed a statement to the effect that it is now "possible for high school vocational or industrial graduates to enroll at more than a score of Michigan colleges and study liberal arts, technical, industrial, and other courses" was misleading its readers. You may enter college, you may take many courses, but there are many other courses and curricula that are almost completely closed to you if you have not had high school mathematics.

For example, Dr. Carrothers of the University of Michigan Bureau of Cooperation with Educational Institutions writes of two girls who were refused admission to the school of nurses of a midwestern university because, though they had the required majors, minors, and entrance units, they did not have high school algebra and geometry.[3]

[2] Your teacher may have a copy of "The Guidance Report of the Commission on Postwar Plans" of the National Council of Teachers of Mathematics which appeared in the November, 1947 issue of THE MATHEMATICS TEACHER. If not you may get one by sending 25 cents to The National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D. C. If ten or more copies are ordered, the price is only ten cents each.

Other sources of similar information are:

Why Study Mathematics. Prepared for the use of teachers and students by the Canadian Mathematical Congress and available from it at Engineering Building, McGill University, Montreal, Canada, for 50¢ each with 20% off on orders for 50 or more.

Professional Opportunities in Mathematics. This 24 page pamphlet is a reprint of an article which appeared in the January, 1951, issue of the *American Mathematical Monthly.* It may be obtained for 25¢ each or 10 for $1.00 from the Mathematical Association of America, University of Buffalo, Buffalo 14, N. Y. It discusses, chiefly, opportunities for persons with advanced training in mathematics (A.B. or more). Such a general knowledge of the importance of, need for, and increasing opportunities for trained mathematicians should be possessed by teachers and guidance persons, but the pamphlet was not written for younger high school students as was the first one listed above. The bibliography of 30 items is extremely helpful for anyone wishing to read in this field.

The Outlook for Women in Mathematics and Statistics. Bulletin No. 223-4, U. S. Department of Labor, Women's Bureau, available from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. 10¢.

[3] George E. Carrothers, "Attitude," *Observa-*

Another school of nursing does *not* specifically require high school mathematics of entering students. Students entering this school may get a "diploma in nursing" in three years, but those wishing a college "degree" have to meet additional requirements which include a chemistry course which in turn requires a year of high school algebra. Note again that before you can be sure you don't need mathematics you must investigate the details, not the mere entrance requirements, of any college program in which you may be interested.

A second example is that of a college senior, a psychology major, who found herself unable to complete her work in mental measurements, the field in which she had wanted to specialize, because the advanced psychology courses required a knowledge of freshman college mathematics. She was lucky. She had taken a year of algebra and a year of geometry in high school. Accordingly she was able to take the required freshman mathematics course and complete her work in psychology. In many colleges you cannot *begin* mathematics. In such schools you must already have had some high school mathematics in order to be able to take even the most elementary mathematics course offered. In other schools, beginning courses are taught for persons who failed to take them in high school, but they do not carry credit toward college graduation. People taking such non-credit courses very often have to spend extra time and extra money to get a college degree.

What, then, should you do to avoid such troubles? If there is any possibility that you might want to go to college, please do this:

1. Write for, obtain, and then carefully study the programs offered at the particular colleges that you might attend. Don't look only at general, formal entrance requirements. Look up the descriptions of the particular courses that you might take.

2. Study the following summary, but remember that requirements differ from college to college. This summary should suggest things to look for in the catalogue of the college you wish to attend, but it is not a substitute for step 1 above.

3. If you have started to plan too late, don't despair. If you need high school mathematics that you don't have, you may: (a) select a college which does teach high school mathematics (but remember that such courses don't usually carry college credit), (b) take a post graduate course in high school, (c) take mathematics by correspondence (but check carefully whether or not and from what schools the college you plan to attend will accept credits by correspondence).

The following list of college courses and programs which require some background of high school mathematics gives an indication of the mathematical preparation expected of many prospective college students. This list is compiled from college catalogues and from letters written by college department heads and administrators in Michigan. The pattern is similar throughout much of this country. However, you are urged again to check carefully the catalogues of the particular colleges which you might attend. For example there may be a Basic Knowledge requirement (as at the University of Illinois, College of Liberal Arts and Sciences) that all students without at least two units credit in high school mathematics must take two semesters of mathematics in college. (At the University of Illinois special courses are offered for these students.)

We have not listed courses and programs when the people in charge have indicated that they believe mathematics is important and provides good mental training for their prospective students, but at the same time they admit that it is

tions. Bureau of Cooperation with Educational Institutions, University of Michigan, Leaflet No. 42, September, 1947.

not required nor even strongly recommended in their institution. We agree with these people that mathematics is important, and we are pleased that they agree with us, but we are here "leaning over backwards" to avoid any idea of spreading propaganda for mathematics. The following are bare facts about minimum requirements in the colleges which were included in this study.

ACTUARIAL MATHEMATICS: See MATHEMATICS.

AERONAUTICS: Elementary, terminal, junior college courses require algebra and geometry.[4] For aeronautical engineering see ENGINEERING.

AGRICULTURE: Algebra and geometry are often required.

ARCHITECTURE; LANDSCAPE ARCHITECTURE: See also DESIGN. Algebra and geometry at least are required by most schools while trigonometry[5] and solid geometry will be required in college (perhaps without "credit") if not completed in high school.

ASTRONOMY: See also SCIENCE AND ARTS: Although "descriptive" astronomy courses may be taken at some schools without even a knowledge of algebra and geometry, such "descriptive" courses often do not permit a student to take any advanced courses nor to specialize in astronomy. Specialization requires mathematics through calculus at least.

BACTERIOLOGY AND BIOLOGICAL CHEMISTRY: High school algebra may be required before taking the chemistry which is often a prerequisite for even the first course in bacteriology.

BOTANY: See also SCIENCE AND ARTS CURRICULUM. Although only one Michigan college requires any mathematics (algebra) for this study, nevertheless, for the study of plant physiology, trigonometry, solid geometry and college algebra (which latter would require high school algebra and geometry) are recommended; for the study of plant genetics, a knowledge of calculus and statistics is recommended at the University of Michigan.

BUSINESS: Algebra or algebra and geometry are required for the Business or Business Administration curricula offered at a number of colleges. At many, all students in Business Administration must take a course in statistics. In one school the prerequisites for statistics are either "adequate mathematical preparation" or a "remedial" course taught in the school. "Adequate mathematical preparation" is interpreted to mean at least one semester of college mathematics, and high school algebra and geometry are assumed to have been studied by all who take even the "remedial" course.

CHEMISTRY: See also BACTERIOLOGY, and SCIENCE AND ARTS CURRICULUM. Algebra or algebra and geometry are often required before taking beginning courses. Advanced work in chemistry must often be accompanied by advanced work in mathematics which in turn is built upon algebra, geometry, and even trigonometry.

CONSERVATION: See FORESTRY.

DENTISTRY AND PRE-DENTAL CURRICULA: Some schools require algebra or geometry. Trigonometry is indirectly required by those colleges that include in the dental curriculum a physics course which in turn requires trigonometry.

DESIGN: See also ARCHITECTURE. Algebra, plane and solid geometry, and trigonometry are required in some schools for the curricula in Interior Design, Product Design, Design of Information Mediums. The non-professional General Design curriculum often has no mathematics requirement.

ECONOMICS: No mathematics is generally required for the beginning courses in this field. Some schools recommend a year of college mathematics for persons majoring in the field and specific advanced courses such as Economic Statistics and

[4] Unless otherwise stated "algebra" means at least a year of high school algebra, and "geometry" means a year of plane geometry.

[5] Wherever trigonometry is required, algebra and geometry must have preceded it whether specifically so stated or not. Trigonometry itself may be studied in high school or college.

Accounting may make definite demands on one's mathematical training.

EDUCATION OR TEACHING: No general statements can be made. Mathematics will be needed in preparing to teach in many fields. For such needs, see the fields in which you are interested. Some advanced courses in psychology, sociology, and education require a knowledge of statistical procedures.

ENGINEERING AND PRE-ENGINEERING CURRICULA: This includes courses in mechanical drawing, machine shop, surveying offered at the college level. Three semesters of algebra, two of geometry and, if possible, one each of solid geometry and trigonometry are minimum requirements. Some schools accept less than this for entrance requiring students to make up their deficiencies without receiving college credit for this work. Others are encouraging high schools to prepare engineers to begin at a more advanced level and are so adding to their curricula that students who do not also have advanced algebra upon entrance must attend an extra term to complete their college work.

ENTOMOLOGY: See SCIENCE AND ARTS.

FORESTRY AND CONSERVATION: Trigonometry, based upon high school algebra and geometry is essential to work in this field.

GEOLOGY: At some schools trigonometry is required for a degree in geology though many courses may be taken without it. Some advanced courses such as geophysics require a knowledge of college mathematics through calculus.

HOME ECONOMICS: In this field, requirements vary from merely a year of mathematics to algebra and geometry.

JOURNALISM: See the note under "Class I" in the latter portion of this paper which summarizes an Indiana Study.

LABORATORY TECHNICIAN: See MEDICAL TECHNOLOGY.

LANDSCAPE ARCHITECTURE: See ARCHITECTURE.

LAW AND PRE-LAW CURRICULA: Very few schools require mathematics, but several recommend a course in mathematics of finance which would need a preliminary knowledge of algebra.

LITERARY CURRICULUM: Few colleges require mathematics for graduation from this program, but at least one requires a year of either mathematics or philosophy.

MACHINE SHOP: See ENGINEERING.

MATHEMATICS: It seems clear that anyone interested in mathematics, statistics, actuarial work or physical science will need and take all of the mathematics available in the high school. Hence, under this heading the important question is not what you should take, but the minimum amount of high school mathematics which you must have in order to be able to take college mathematics for credit.

If you have less than a year of algebra and one of geometry, you may in some colleges take these elementary high school courses in college, but often without college credit. In other colleges no beginning algebra and geometry courses are taught.

MECHANICAL DRAWING: See ENGINEERING.

MEDICAL TECHNOLOGY: Algebra, algebra and geometry, two years of algebra and trigonometry are variously required on this program.

MEDICINE AND PRE-MEDICAL CURRICULA: See also PUBLIC HEALTH. Algebra, algebra and geometry, and trigonometry are required at various schools. Trigonometry is indirectly required by those schools which require college physics which in turn demands trigonometry.

MINISTRY: See the note under Class I in the latter portion of this paper which summarizes an Indiana study.

NURSING: Algebra and a second year of some high school mathematics are required at one school. Algebra is a prerequisite for the college chemistry which is required for a *degree* in nursing at another. A *diploma* in nursing may have no mathematical requirements.

PHARMACY AND PRE-PHARMACY CURRICULA: Algebra and geometry are required at most schools.

PHILOSOPHY: For advanced (graduate) work in philosophy and logic, mathematics even beyond calculus is recommended. To take such college courses one must have had high school algebra and geometry. These courses are required at a few colleges.

PHYSICS: See also SCIENCE AND ARTS. Algebra and geometry are commonly required with some schools adding trigonometry which can, however, be taken in college.

POLICE ADMINISTRATION: Algebra is prerequisite to this program at one school.

POLITICAL SCIENCE: Mathematics is particularly desirable only for advanced work in branches that deal with statistics or governmental accounting. At one college a "Public Service" curriculum is included with their Business program which in turn requires algebra and geometry.

PSYCHOLOGY: In general, algebra is required only for advanced courses in mental measurements, statistics and experimental vocational psychology. However, students wishing to "concentrate" in this department at one school must take "Psychology 40. Elementary Statistical Methods in Psychology" for which a prerequisite is "satisfactory performance on a qualifying examination in elementary mathematics (equivalent to the algebra content of Math 7.)" Algebra and geometry are prerequisite for "Math 7" in this school.

PUBLIC HEALTH: Work in this field may require a knowledge of chemistry, medicine, or nursing. See the requirements for these subjects and the catalogue of the school of public health which you wish to attend. Statistical procedures are utilized in several Public Health curricula.

PUBLIC SERVICE: See POLITICAL SCIENCE.

SCIENCE AND ARTS CURRICULUM: Algebra and geometry are required at one school on this curriculum which includes all students who seek a bachelor's degree with a major in Bacteriology, Botany, Chemistry, Entomology, Geology, Mathematics, Physics and Astronomy, or Zoology. See the specific sciences in which you are interested for the requirements at other colleges.

SOCIAL SERVICE: At one school this curriculum is grouped with Business and Public Service. See these headings, also SOCIOLOGY.

SOCIOLOGY: See also SOCIAL SERVICE. Algebra and geometry are recommended only as they are necessary prior to taking the college mathematics which is desirable for persons interested in population problems and statistical research in social problems. At one college all students who "concentrate" in Sociology must take "Sociology 90. Introductory Quantitative Sociology." Although this course has no mathematical prerequisite, the instructor strongly recommends algebra, geometry, trigonometry, and college algebra.

STATISTICS: Course work in statistics may be given in the Mathematics Department, in a separate Statistics Department, or in specialized courses in other departments such as Economics, Education, Psychology and Sociology. In the first case the prerequisites are usually those of the Mathematics Department. In the latter cases the requirements are listed under the particular departments.

SURVEYING: See ENGINEERING.

VETERINARY MEDICINE: Algebra and geometry are required of students in this field.

ZOOLOGY: See also SCIENCE AND ARTS. There is no general mathematics requirement in this field, however algebra is required at one school and is a prerequisite for the chemistry which zoology "concentrates" must take at another. Trigonometry, college algebra, and even more advanced courses, especially statistics, are recommended for persons preparing for *research* in zoology, natural history, wildlife, ichthyology.

The above curriculum requirements are based upon a study of Michigan colleges; the following classification is based upon data gathered through personal interviews

with department heads in thirty colleges in Indiana enrolling nearly sixty thousand students.[6] The department heads were questioned on the actual mathematical needs of students in their field irrespective of the formal entrance requirements of their college. This classification should be of particular value for the high school students who plan to go to college, who have a general idea of their interests, but who have not decided upon a specific course of study.

None of the schools cooperating in this study report that first year (high school) algebra or plane geometry is offered for college credit. A few report that such courses may be taken without credit in an affiliated high school. Several schools offer intermediate algebra and solid geometry as an elective but do not count them on the major in mathematics. In most cases the courses which count toward a *major* in mathematics presuppose three semesters of high school algebra and two semesters of plane geometry.

On the basis of their need for preparation in high school mathematics most Indiana college departments fall rather easily into three classes. The first class includes those fields of study in which mathematics is not a prerequisite. The second class includes those departments in which some college mathematics is required and which therefore demand of the student a preparation in high school mathematics sufficient to enable him to pursue the study of college mathematics. The third class includes those fields of study which do not demand a knowledge of college mathematics directly but which do demand some knowledge of another field in which some mathematics is required.

[6] This study was sponsored by the Indiana Section of the Mathematical Association of America and was carried out by a committee consisting of K. P. Williams, Indiana University; H. F. S. Jonah, Purdue University; and P. D. Edwards (Chairman) of Ball State Teachers College. A preliminary survey of its implications may be found in P. D. Edwards' "Minimum Mathematical Preparation for Various College Curricula," *School Science and Mathematics*, XLIX (March 1949), 181–187.

Class I. *Mathematics not a prerequisite* (but note the *exceptions!*)

In most instances no mathematics is required for the study of ENGLISH, FOREIGN LANGUAGE, HISTORY, THEOLOGY, MUSIC, ART, PHYSICAL EDUCATION, and JOURNALISM. The exceptions are worthy of comment. In one institution approximately forty per cent of the students of JOURNALISM prepare to become publishers and include in their courses of study, courses in accounting and in statistics. One denominational school requires students preparing for the MINISTRY to include studies in business and in church architecture. Several departments of ART recommend the study of geometry. Three institutions offer degrees in LAW. In each case no specific requirements were listed in mathematics but in each case the entrance requirements were those of the School of Arts and Sciences and include a minimum of one year of algebra and one year of geometry.

Class II. *College mathematics required*

Departments in which courses in mathematics are required include all branches of ENGINEERING, PHYSICS, PHYSICAL CHEMISTRY, ASTRONOMY, METEOROLOGY, and GEOLOGY. Only one institution offers a degree in FORESTRY; it has college mathematics as a requirement. In several institutions mathematics is listed as a requirement of all majors in CHEMISTRY. In others, high school algebra through fractional exponents and logarithms is required and geometry is listed as desirable. It may probably be assumed that these differences in replies reflect the different standards for the various institutions. These differences in the requirements in chemistry are especially interesting in view of the universal requirement of some knowledge of chemistry for HOME ECONOMICS and NURSING.

On the graduate level, and in some institutions on the undergraduate level, a

knowledge of college algebra and statistics is required for students of SOCIOLOGY, ECONOMICS, BUSINESS ADMINISTRATION, and BIOLOGY. The requirement, however, is not uniform and again it seems to reflect the standard of the institution, or the particular field of specialization.

Class III. *Indirect requirement*

Several professions make no specific requirement of mathematics but list prerequisites which in turn require a knowledge of some college mathematics or at least of a minimum of three semesters of high school algebra and two semesters of geometry. This requirement arises most frequently through a requirement in physics and chemistry. Into this class fall MEDICINE, DENTISTRY, MEDICAL TECHNICIAN, NURSING, and PHARMACY.

There are a few departments that are not easily placed in any one class. A good example is the SPEECH DEPARTMENT which is found in several Indiana colleges. In some colleges the activity of this department is restricted to work in Public Speaking and Debating with no requirement in mathematics. In other institutions the Speech Department includes the clinical study of speech abnormalities. This work borders on the field of medicine and preparation is required in physics and anatomy. A similar variation is found in the field of DRAMATICS. Generally mathematics is not required. In one report, however, it is stated that the course in dramatics includes stage lighting and sound amplification, and that courses in physics are required.

This is the end of our "message" to you high-schoolers. Our advice to you is as follows:

(1) Check the details of the programs in which you are interested in the catalogues of the colleges which you might attend before you decide that high school mathematics is unnecessary for your future.

(2) At least a year of algebra and a year of geometry (or their equivalent in a well planned two year sequence of high school mathematics) is a pretty sound investment for all college preparatory students.

Such an investment of time is sound insurance against later loss of time and embarrassment. High school and even college students often change their minds as to the curriculum or college of their choice. Without the insurance of some high school mathematical work they may find to their embarrassment that their new choice has an unexpected requirement which it is difficult or impossible to satisfy.

To *high school teachers, guidance officers,* and *administrators* we should like to emphasize that this article is concerned only with the mathematical needs of college preparatory students and moreover, as we shall emphasize shortly, it does not preclude the possibility of the reorganization of the traditional courses into a more effective sequence of more generally valuable topics. Further, we feel that just as advisors have a serious responsibility to call these facts to the attention of students, and as administrators owe it to both the students and the nation to provide time for and competent instruction in substantial mathematics work in high school, so do these two groups with the mathematics teachers have a responsibility to criticize, reorganize, improve, and invigorate the content, materials and methods used in mathematics instruction.

We have presented above a list of minimum mathematical requirements that are similar to those any prospective college student should expect to find in the college and subject of his choice. This list is not concerned with the requirements for gaining admission to college. It is concerned with the needs of students if they are to be able to continue effectively their training in college without unnecessary (and frequently embarrassing) confusion, and extra precious years of youth and money spent on removing deficiencies. The effectiveness of this list depends entirely upon this information being made available to

every prospective college student *early in his high school career*. We cannot emphasize too strongly that early in his high school career each college preparatory student needs an opportunity to know and to discuss with competent advisors his own mathematical needs in his (at least tentatively) chosen life work. This knowledge of their own personal mathematical needs should be a source of considerable motivation for college preparatory students.

It is easy to substantiate the fact that our civilization is becoming more mathematical at all levels and in all fields at a rate never before known. It is paradoxical that at such a time some persons have interpreted current educational philosophies to imply that *less* mathematics should be taught when the real need is for *more* mathematics. On the other hand, in many schools the selection of topics and instruction could be materially improved. Some of these desired improvements relate to mathematics for general education and will not be discussed here. Other possible changes may be suggested by the following paragraphs which are also addressed to college staffs.

College staffs in mathematics and other fields should realize that originally they phrased requirements in terms of *courses* which were commonly offered in the high schools. Many high schools now believe, with some justification, that they must teach traditional "algebra" and "geometry" *courses* because colleges require them. This has led to inconsistencies as well as to misunderstandings. For example, one college makes a year of geometry a prerequisite to chemistry, but the mathematics needed in their chemistry course is largely ratio and proportion, not geometry. It is true that some drill in proportion is found in demonstrative geometry as well as some practice in abstract and logical thinking which may or may not "transfer" to later problem situations. However, were there better understanding and cooperation between high schools and colleges the schools might design a course which would be better for pre-chemistry students as well as all others. Similarly another chemistry department requires a year of high school algebra because they wish students to be familiar with the theory of logarithms and fractional and negative exponents. They then complain of the students they get, blissfully unaware that few ninth grade algebra courses cover these topics.

The need is for a more realistic and specific recognition of not only the fact that mathematics is essential, but also of *what* mathematics is essential *where*. The University of Illinois has taken several significant steps in this direction. The College of Engineering at this institution explicitly recognizes that in many fields modern engineering has advanced in both theory and practice to the point where even basic undergraduate training may require more than four years unless some means of presenting additional material can be found. With these facts in mind this college has gone on record as requiring (effective September 1953) that all students who wish to enter without deficiencies must have a mastery of the topics usually found in four years of high school mathematics. Students entering with less than this must in general expect to put in extra time in college to make up these deficiencies.

The above requirement has been proved to be realistic. Twenty-three high schools in Illinois now offer courses in algebra and trigonometry that are recognized by the University of Illinois as equivalent to those taught on the university campus. Other high schools are encouraged to make it possible for their students to begin their college mathematics with analytic geometry. To this end a committee at the University of Illinois has drawn up a list of the minimum mathematical needs of prospective engineering college freshman after carefully studying the problem and existing related research. Although this list was specifically compiled to make known to high school students and teachers the minimum mathematical needs of prospective students at the University of Illinois College of Engineering, the com-

mittee believes that the two parts of this list are applicable to all students planning to start their college mathematical training with (1) college algebra and trigonometry or (2) analytic geometry.[7]

The list compiled at the University of Illinois may be used in any high school as a basis for organizing the mathematics courses so that high school graduates should, possibly by taking an appropriate placement test, be able to begin college mathematics with analytic geometry. Under the above program powerful mathematical tools are available early in the student's college career and may be used very effectively in physics and other courses. This program is primarily designed to enable engineering students to start their college work at a more advanced level in order to avoid spending more than four years as an undergraduate. It can also be highly advantageous to prospective college students in any of the many curricula requiring college algebra and trigonometry in college. In such cases the student should be able to take additional elective college courses either in his major field or in a field of special interest to him.

All colleges might use this or a similar list to appraise their entrance requirements. For example, since the principal concern of the colleges is the material mastered, the entrance requirements might be stated in a form such as: three years of high school mathematics in preparation for college algebra [four years in preparation for analytic geometry] with competence relative to certain listed minimum mathematical needs. This would allow the high school teachers considerable freedom in both the organization and the selection of the material to be presented and still provide a designation of the goals to be attained. This use of carefully analyzed specific requirements would facilitate the growth of experimental integrated mathematical courses such as are now being tried in New York State. Other secondary schools might then be able to organize two years of general (unified) mathematics that will have broad appeal and which may be used in working toward college algebra either in high school or college.

Collegiate mathematicians should also realize that there are very important fields which make a minimum demand for mathematical training. In the fields covered by the Language Arts, Music, Art, Theology, History, Government, Journalism, and Physical Education one finds most of the courses that contribute directly to the training of writers, public speakers, and entertainers. Briefly these are the very persons who do most to shape public opinion. However important mathematical training may be in modern civilization, the fact remains that the very persons who do most to influence mass thinking are the same persons who require and receive the least preparation in technical mathematics. Thus increased attention must be given to the planning and teaching of collegiate mathematics courses designed for general education as well as to the improvement from the viewpoint of general education of existing traditional courses.

In any case, the need seems clear for cooperation between high schools and colleges in making clear to students the role of mathematics in today's world and in then facing the responsibility for providing sound and even inspiring teaching in a mathematics curriculum which is realistically and not merely traditionally geared to the needs of both the student and the nation.

[7] The conclusions of this committee [Professors R. P. Hoelscher and M. O. Schmidt from the College of Engineering, W. A. Ferguson and B. E. Meserve from the Department of Mathematics, K. B. Henderson (Chairman) and Mr. K. W. Dickman from the College of Education] are stated in the University of Illinois Bulletin, Vol. 49, no. 9, *Mathematical Needs of Prospective Students in the College of Engineering of the University of Illinois*, September 1951, Urbana, Illinois.

A further discussion of the work of this committee appears in THE MATHEMATICS TEACHER, XLV (February 1952), 89–93.

Mathematics in Engineering Research*

By Chas. M. Cooper
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The subject for this evening's talk no doubt calls to mind such things as vector analysis, wave mechanics, Bessel functions, differential equations, matrix algebra, analog computers, and the theory of relativity. I hope you will not be disappointed if such subjects, important though they be, are not mentioned again this evening. I have no intention of belittling these tools of higher mathematics and of theoretical physics. They have their place and it's an important one; but, in engineering research as in most other phases of living, most progress toward understanding is made through clear, quantitative thinking, employing the simpler mathematical tools taught in elementary and high schools and in the first year or two of college work. So it will be arithmetic rather than analog computers this evening.

I have been privileged for over twenty-five years to observe many engineers in action and to note the wide variation in effectiveness among people who have had the same formal schooling. Some people grow much more rapidly in effectiveness and judgment than others. Why? What do they have that is lacking in others? The complete answer would surely be of utmost interest to all educators but it's not likely to be fully available in the near future. I wish to suggest to you tonight that part of the difference may lie in the degree to which the individual practices on all occasions *quantitative thinking.* I will attempt to show you that *quantitative thinking* is of necessity the basis of all engineering research, and that our daily national life would benefit if a larger fraction of our population practiced it. *Quantitative thinking* implies the use of mathematical tools and indeed can be engaged in easily only when the mathematical operations involved have become second nature. And so, I will finally get back to you, the teachers of mathematics, and suggest to you—strictly as an amateur of course—what you may do to help the cause.

In spite of the positive sound of the remarks up to this point, it is really with some diffidence that I broach this matter to you. It probably is far from a new thought, and may be well recognized already by teachers of mathematics.

Perhaps at this point it would be well to note that I have never taught mathematics (my attempts to help my daughters with their school work met with, shall we say, indifferent success), and I narrowly escaped flunking a course or two. Consequently, I should be well prepared, don't you think to lecture teachers of mathematics?

Before attempting to pin down this subject of "quantitative thinking" by a cold and formal definition, I'd like to tell you a story. Enrico Fermi and Arthur Compton, both Nobel prize winners in physics, were on their way by train from Chicago to the Hanford Plant on the Columbia River. Compton said to Fermi, "Enrico, for three days we will be traveling at an elevation perhaps 4000 feet above Chicago. Will the change in elevation have any effect on the timekeeping of our watches?" Fermi thought a moment, pulled out his six-inch slide rule for a few passes and said, "Yes,—fast—not more than ten seconds." Pure magic what? But then you would expect a famous physicist to know things you and I wouldn't be expected to remember. Actually that's not the answer in this case. Fermi had said to himself something like

* Presented at the dinner meeting of the 11th Annual Institute for Teachers of Mathematics, Duke University, August 11, 1951.

this. "If the timekeeping changes, it will be because the effective mass of the balance wheel changes. I suppose some air moves back and forth with the balance wheel and thereby increases its effective mass. At higher altitudes less air will be involved, the effective mass of the wheel will be less and it can go faster. Of course, I've no idea how much air will be associated with the wheel, but certainly less than one volume of air per volume of balance wheel. To get an idea of the size of things, what would one volume per volume amount to? One cubic centimeter of air will weigh about 29 divided by 22,400 (the molecular weight of air divided by the standard molecular volume) or say one-thousandth of a gram. One cubic centimeter of brass might weigh about 8 grams. Hence, an equal volume of air would weigh about one eight-thousandths as much as the wheel. A change in the effective mass of this amount, if the effect is linear, would have corresponding effect on time. There are 3600 seconds in an hour. Hence, speed-up in the order of $\frac{1}{2}$ second per hour or 36 seconds for the three days would result. However, an altitude change of 4000 feet would change the atmospheric pressure only a small fraction. Hence, it's safe to say that the speed-up will amount to less than 10 seconds." That locates the effect as small enough to forget for ordinary purposes, but if an error of no greater than a second per day or less is required for some purpose, we had better take a closer look. Most of us, I suspect, had we analyzed the problem in the first place, would have thrown up our hands saying, "The effect will be small and it would take a very careful experimental study to define it." We would remember that a watch will run fast. Fermi remembers it will run fast but certainly less than 3 seconds per day from this cause alone. Is it too much to suggest that somewhere herein may lie the reason that Fermi is a top-ranking physicist? In my experience Fermi is tops in quantitative thinking. Since every question gets similar treatment, he continually is using information already in his mind to build up further facts. This process of self-education is almost automatic —and very effective.

Perhaps, with this story, you begin to see what quantitative thinking implies to me. The person who carries it to the extreme will in every situation that has quantitative aspects, almost automatically estimate the size of these factors to the extent possible with the time and data immediately available. Visiting a paper mill he sees a log about a foot in diameter and four feet long going into the process. How many newspapers will that make? Let's see—3 cubic feet at, say, 40 pounds per cubic foot—about 120 pounds. At least half, or 60 pounds might end up as paper. A large newspaper weighs perhaps $\frac{1}{2}$ pound, so I'm looking at 120 newspapers. Or his thoughts may take a different course. The log if sawed into boards might net 2 cubic feet or, say, 20 board feet worth perhaps 10 cents per board foot, or $2.00. Two dollars for 60 pounds would be about $70 per ton. Hence, paper must cost more than $70 per ton to make.

Now let's get back to "Mathematics in Engineering Research" and see where this matter of quantitative thinking fits, if at all. On one occasion for test purposes we wanted to compress a gas rapidly by means of a fast moving piston in a steel cylinder. We could put a known amount of energy—about 20,000 foot pounds—into the piston. The question was—how much of the initial energy could be expected to be stored in the compressed gas at the end of the stroke? This turned out to be a most involved problem because energy would be lost as mechanical friction, as heat from the hot gas, and as leakage around the piston. Measurements showed that only 60 to 70 per cent of the original piston energy was in the gas at the end of the stroke. The loss, therefore, amounted to, say, 6000 foot pounds. Why? Measurement of the individual losses would be most difficult. How could you calculate anything? The coefficient of

friction between piston and wall was completely unknown; the mechanism of heat transfer from the hot gas to the wall could hardly be unscrambled; even the clearance between piston and cylinder would vary so much during the cycle because of the high final pressures that exact calculation of leakage was not possible. Sort of hopeless? Not at all. Let's look at friction. The coefficient, while unknown, can't be greater than 1.0. The piston weighed 4 pounds—hence a push of 4 pounds was the most that could be required to push it. With a three-foot stroke we could use up in mechanical friction no more than 4×3 or 12 foot pounds. The greatest possible loss to friction would then be less than 0.2% of the total measured loss, and quite unimportant. Heat losses? We considered the various ways in which heat might be lost and picked the worst possible case. Similar mental calculations showed that this "worst" loss could not be more than 3%. How much less we didn't know, but 3% isn't important in the face of 30 to 40%. Leakage past the piston? The clearance as we said, was unknown, but it could hardly be more than two thousandths or less than 0.5 thousandths of an inch, corresponding to energy losses between 50% and 15%. Clearly, this was the source of the loss. And it took about twenty minutes to make these estimates. Pretty obvious, you say. Anyone should be expected to do that. I think so too, but it might surprise you how frustrated most college graduates become when faced with this kind of a problem. They are prepared to calculate laboriously, if necessary, the exact answer provided data are available, but they are totally unprepared to develop quickly the *size* of the problem, using readily available information. You can see how important it is to know the size of each problem element so you spend your time on the important ones. Note especially the starting point for the leakage calculation. It was known that the low pressure clearance would be 0.5 thousandths of an inch, and it was easy to show that at high pressure it could not be greater than 2 thousandths, but must be greater than the low pressure value. Hence, the actual value *had to lie between the two values.* Had it been worthwhile, a similar bracketing could have been applied to the friction and the heat loss.

Engineering research deals with ideas. Some one has said that "if a man has a batting average of 1% on his ideas, he is good." The man with ideas must have a screen or the chances are he will waste his time on a poor idea and never recognize the good one. I know just such a man. He has a most fertile imagination and he does a beautiful job of calculating up ideas, but he wastes most of his time and the time of others on poor ideas.

Engineering research deals with many, many calculations; and engineers are as prone as others to decimal point errors. Such errors never get by the man who thinks quantitatively because he knows the general size of the answer before he starts detailed calculations. I know of a man now in his forties, well trained as an engineer and with a lot of experience, but he has no "feel" for size and his calculations are untrustworthy.

Yes, engineering research hangs heavily upon quantitative thinking. Let's summarize some of the reasons. The daily practice of quantitative thinking—

1. Provides a process of self-education which daily stores up new information in the mind.
2. Serves to make available information already stored in the mind.
3. Rapidly provides a "feel" for a new field.
4. Provides for screening the important from the unimportant.
5. Gives the man with ideas a chance to screen rapidly and pick out the good ones.
6. Makes decimal point errors unlikely.
7. Rapidly provides the basis for judgment.
8. Finally, and perhaps most importantly in the long run, satisfies curios-

ity and serves to stimulate the imagination.

No doubt as this talk has gone along you all have seen the application to all aspects of national and personal life. To understand the significance of our modern economic age, one needs constantly to indulge in quantitative thinking. I wonder how many people ever estimate their share of the national tax burden. Let's see. I don't remember the current figure discussed in Congress but it will be different tomorrow, so let's take forty-five billion for this illustration. Forty-five billion dollars divided by, say, one hundred and fifty million people gives around three hundred dollars per person, or for an average family of four people, say, twelve hundred dollars. Each time another ten billion is added the bill goes up another two hundred and forty dollars. Of course, most of us do not pay any such direct income tax, and without some honest quantitative thinking we fail to realize the size of our indirect tax bill. If we all realized how much we pay in higher prices for "free" government services, we might begin to question whether of our own free will we would buy the services. I really wonder how many people ever attempt such simple but important calculations. I don't believe the fraction is as large as one in a hundred.

I promised that I would come back at you before finishing with some ideas, strictly as an amateur, for you as teachers of mathematics. The same suggestions could apply with equal merit to teachers of science and of economics—maybe others. Teachers of mathematics, however, have the first chance at the student.

You will have noticed that the examples used so far have required nothing but arithmetic, and that is proper, for arithmetic is by all odds the most used branch of mathematics. Now I, personally, had no real trouble with mathematics while in school. On some occasions I could have applied myself more actively though my record was possibly better than average. But I did not learn to add in that little red country school house near the Hudson River. In 1921, after a year at Massachusetts Institute of Technology, I found myself somewhat unexpectedly engineer, timekeeper, and paymaster of a small construction operation—about thirty employees—in Maine's Penobscot Bay. There I learned to add.

The first time I made out the payroll it took two frustrated days to balance it, but before the summer ended, it wasn't bad. I had learned to add, you see, but not really well. Some years later I built a house, and kept careful records of all expenses. I even prepared summaries of costs under the headings: electrical, plumbing, heating, etc., but to this day I don't know exactly what the house cost or how the cost broke down, because those long columns to be added up truly discouraged me! Pitiful, isn't it? If only I had practiced addition sufficiently at an early age I might not have that mental hurdle to get over. Addition is a tool in much the same sense as a carpenter's plane. You can't learn to make a true, flat surface simply by reading about the use of a plane. You have to use it.

I don't believe there is any way to learn to add with the same kind of facility as youngsters achieve in, say, skipping rope, except by practice. And unless one can use his mathematics with as little effort and thought as is involved in walking, unless, in fact, its use is essentially automatic, there will exist a mental hurdle which will keep the individual from thinking quantitatively. On the other hand, mental mathematics can be, as you well know, a game. Note, however, that the goal toward which I am pushing you is more than mental calculation. Let's call it "sizing the problem." Suppose that from earliest arithmetic every problem worked out in class is handled in two parts. First, by inspection, what is the approximate answer? Next, what is the exact answer? For instance, multiply 22 by 37. The answer will be close to 20 times 40 or 800. Extend this approach from mathematics to chemistry,

physics, economics, and all engineering courses. Form early the *habit* of sizing all problems. Quantitative thinking, however, must go farther than simple sizing. To be effective the user must learn to draw his data from his own mental storehouse. This requires practice too—and is even more fun than simple mental calculation. For example—you are sitting lazily in the hammock thinking about possible vacation plans and you wonder idly how much it would cost to visit the Philippine Islands by plane. Let's see. The earth is about 24,000 miles in circumference so two places cannot be more than 12,000 miles apart. Manila is pretty well on the other side, say 10,000 miles. You remember that first class railroad fare with meals and tax runs around six cents per mile, and you know that in the United States plane fares are competitive with trains. However, much of the trip will be over water where there is no competition from trains. The fare certainly won't be less than six cents per mile and hardly twice as much. Say nine cents a mile. One way, $900. "No need for more exact figures," you say. "It'll be a long time before I take *that* trip." (After making this estimate, I called the air line for information—10,380 miles, $960.)

Perhaps the most distinctive feature of the human race is inertia. Physical inertia is readily recognized. Mental inertia, while not so obvious, is at least as prevalent. It's such an effort to decide to do anything! If you want to badly enough, you'll make the effort. At the age of ten it's an effort to wash behind the ears—unless there's a movie or some other driving influence. Do you remember the story about the tramp who asked a farmer "could he work for his dinner?" The farmer was planting his garden so he put the tramp to cutting up seed potatoes. Some of you may have performed this operation. The resulting pieces should be chunky rather than slices and each piece should contain at least one eye—yes, potatoes have eyes. Some time later the tramp came up to the farmer and said, "Boss, ain't you got something easier? I just can't make all those decisions." You see a pure case of mental inertia. Maybe he wasn't hungry enough.

There are two ways to get around the mental inertia problem. You can increase the interest by prospect of reward which may run the gamut from ice cream sodas to mental satisfaction; or you can make it easier to accomplish the desired end. Grease the skids so to speak. Habits are little helpers in this regard. When you get up in the morning do you consciously decide which shoe goes on first, or whether to shave and then get dressed? Just think of the hundred decisions between shutting off the alarm clock and drinking the last sip of coffee. Why, just contemplating them would be enough to send you back to bed! But those little pathways worn in the brain by all your actions up to the time make it easier to do one thing rather than another. In fact, after some time, it even becomes easier to wash behind your ears than to make a decision and let the dirt lie. So habits are all important when it comes to making easy a path—either bad or good. If we want people to think quantitatively we've got to fix the habit so thoroughly that it's easier to do so than not. But you can't do that unless the processes of elementary mathematics are as easy as walking. And that puts the problem up to you—the teachers of mathematics. I can't help feeling that you can do more to open the eyes and minds of each new generation than teachers in any other field. Not by special courses, but by the way problems are approached, by example. I'm sure that if you yourselves find as much interest in "sizing" each situation as do I, that the students just won't have a chance to avoid the habit of quantitative thinking.

There you have my story. Let me run over it briefly in summary. Quantitative thinking, as I am using the term, is a habit of thought which when acquired causes

(*Continued on page 339*)

September Hath XIX Days

By Vera Sanford
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"Kind Reader,

"Since the King and Parliament have thought fit to alter our Year by taking eleven days out of *September*, 1752, and directing us to begin our Account for the future on the First of *January*, some Account of the Changes the year hath heretofore undergone, and the Reasons of them, may a little gratify thy Curiosity."

Poor Richard's Almanac, 1752

Discussions of calendar reform in 1952 are concerned with the rearrangement of the number of days assigned to the months of the year to make the calendar more convenient for business purposes. This regrouping of the days, however, is within the framework of the calendar as set up by Julius Caesar in 45 B.C. and revised under Pope Gregory XIII in 1582. In 1752, in England and in the colonies calendar reform meant a long overdue shift from the Julian calendar to the Gregorian one. It is the purpose of this article to review briefly the considerations which led to the formulation of the Julian calendar and to its improved form as the Gregorian one, and to then consider in greater detail the change that took place in England in 1752.

The calendar used in Rome when Caesar came to power consisted of a 355 day year with an extra month added from time to time to keep dates on the calendar in line with the seasons. This process had been neglected to such an extent that it is reported that a certain date in the month of August according to the calendar was "toward the end of May by real time."[1] Caesar employed the astronomer Sosigenes of Alexandria to bring the dates in the calendar to conform to the seasons and to plan a calendar which would prevent a recurrence of the difficulty. The new calendar consisted of a cycle of three years of 365 days each and a leap year[2] of 366 days every fourth year. The year was to begin on January 1, and compensate for accumulated errors, the "year of the change" consisted of 455 days with resulting difficulties in the collection of taxes. Benjamin Franklin, under the pseudonym of R. Saunders in the *Almanac* quoted at the beginning of this article, calls this year *Annus Confusionis* or the *Year of Confusion*.

Between the adoption of the Julian calendar and the calendar reform of the sixteenth century, a number of variations arose in practice. The new year was considered as beginning at dates other than January first—England using the Feast of the Annunciation (Lady Day), March 25, as the start of a new year.

The Julian calendar reckoned the year as being $365\frac{1}{4}$ days while in reality it is about 365 days, 5 hours, 48 minutes, 46 seconds long, the error being about $11\frac{1}{4}$ minutes a year. By the last quarter of the fifteenth century, this cumulative error was becoming noticeable and the vernal equinox instead of coming on March 21 came about ten days earlier. Studies regarding the calendar were initiated, but no action was taken.

In the Pontificate of Gregory XIII, however, a group of mathematicians and astronomers staged a demonstration of the discrepancy between the Julian Calendar and the astronomical one. One of the group was Ignazio Dante, a mathematician of Bologna, Gregory's native city. In 1575, Dante had constructed a calendar line in the church of San Petronio in Bo-

[1] James Anthony Froude, *Caesar*. XXII (New York: Scribner, 1879), 387.

[2] According to the *Oxford English Dictionary*, leap year got its name from the fact that fixed festivals coming after February in such years come two days later in the week than in the preceding year, while in ordinary years, they advance by one day only.

logna. At noon, the sun's rays coming through a small window high in the south wall, rested on a north-south line coming closest to the south wall at the summer solstice, and farthest from it at the winter one thus giving optical demonstration that the actual summer solstice came earlier than the supposed date, June 21. This line, rectified by Gian Domenico Cassini, is still to be seen. A similar line was arranged in a room now known as the Sala del Calendario (Calendar Room) in the Tower of the Winds in the Vatican. The demonstration was convincing to the authorities, and in 1582 the calendar was set ahead by ten days in a decree that October 4 should be followed by October 15. This brought the vernal equinox to its assigned place, March 21, this being the date fixed as the vernal equinox in the Council of Nicea (325) which had decided the rule to be followed in the determination of the date of Easter. To preserve this, it was necessary for a day to be omitted three times in every 400 years—the amount by which the Julian calendar was in excess in that period. The adjustment was made by omitting the leap year in century years not divisible by 400. The new year was to begin on January 1. The new way of reckoning the calendar was called the New Style, and was adopted in Roman Catholic countries at once.

A bill to change the calendar in England was proposed in the English Parliament in 1584, but it was withdrawn after a second reading and England continued with the old calendar—the Old Style, and began the year on March 25 as before. The difference of 10 days in 1582 became a difference of 11 days in 1700 when the Gregorian calendar skipped a leap year. (It was at this time that the German Lutheran countries changed to the new calendar.)

The difficulties consequent on the difference in the date on which the year began between England and continental Europe led to the use of "slash dates" in records. Thus George Washington's birth was recorded as 11 February, 1731/32 O.S., meaning the eleventh of February by the Julian calendar in the year that was counted as being 1731 in England but 1732 on the continent. Had he been born in April, the year would have been 1732 by both reckonings.

By the middle of the eighteenth century, the difference in the calendars seemed inexcusable to a number of thoughtful minded Englishmen. Among them was George Parker, Second Earl of Macclesfield (1697–1764). He had been a student of the mathematician Abraham de Moivre and he was a close friend of the astronomer James Bradley. Macclesfield had built an observatory for his own use, and had made many astronomical observations. He seems to have been chiefly responsible for the change in the calendar. At the meeting of the Royal Society in May 1750, Macclesfield presented a paper with the title "Remarks upon the Solar and the Lunar Years."

On the twentieth of February, 1751, a "Bill for Regulating the Commencement of the Year" was introduced into the House of Lords by Philip Dormer Stanhope, Second Earl of Chesterfield. Chesterfield did this in spite of the urging of Lord Newcastle who took the stand that it was unnecessary to bring this controversial subject up at that particular time. On the eighteenth of March, the bill was given a second reading and Macclesfield made a speech in explanation. The bill was passed. Chesterfield described the two occasions in letters to his son dated Feb. 28 and March 18 O.S. from which the following quotations are taken.[3]

> I have of late been a sort of *astronome malgré moi*, by bringing in last Monday into the House of Lords a bill for reforming our present Calendar and taking the New Style. Upon which occasion I was obliged to talk some astronomical jargon, of which I did not understand one word, but got it by heart, and spoke it by rote from a

[3] Philip Dormer Stanhope, Earl of Chesterfield, *Letters to his Son* (Washington and London: M. Walter Dunne, Publisher, 1901), vol. I, 385, 393–4.

master. I wished I had known a little more of it myself.

I acquainted you in a former letter that I had brought a bill into the House of Lords for correcting and reforming our present calendar, which is the Julian, and for adopting the Gregorian. I will now give you a more particular account of that affair: . . . It was notorious, that the Julian calendar was erroneous, and had overcharged the solar year with eleven days. Pope Gregory the Thirteenth corrected this error; his reformed calendar was immediately received by all the Catholic powers of Europe, and afterward adopted by all the Protestant ones, except Russia, Sweden, and England. It was not, in my opinion, very honourable for England to remain in gross and avowed error, especially in such company; the inconveniency of it was likewise felt by all who had foreign correspondences, whether political or mercantile. I determined, therefore, to attempt the reformation; I consulted the best lawyers and the most skillful astronomers, and we cooked up a bill for that purpose. But then my difficulty began: I was to bring in this bill, which was necessarily composed of law jargon and astronomical calculations, to both of which I am an utter stranger. However, it was absolutely necessary to make the House of Lords think that I knew something of the matter; and also to make them believe that they knew something of it themselves, which they do not . . . so I resolved to do better than speak to the purpose, and to please instead of informing them. I gave them, therefore, only an historical account of calendars, from the Egyptian down to the Gregorian, amusing them now and then with little episodes; but I was particularly attentive to the choice of my words, to the harmony and roundness of my periods, to my elocution, to my action . . . many of them said that I had made the whole very clear to them. . . . Lord Macclesfield, who had the greatest share in forming the bill, and who is one of the greatest mathematicians and astronomers in Europe, spoke afterward with infinite knowledge and all the clearness that so intricate a matter would admit of; but as his words, his periods, and his utterance, were not near so good as mine, the preference was most unanimously, though most injustly, given to me. . . .

The bill provided that the English calendar be brought into line with the Gregorian one by dropping out the days from September 2 to September 14, 1752 and by beginning each year with the first day of January. Thus *Poor Richard's Almanac* has the pages for September headed "SEPTEMBER. IX Month" and on the opposite page, "September hath xix Days." On these pages, the calendar for September read September 1, 2, 14, 15, . . . 30, but the days of the week went on without break—the first was Tuesday, the second was Wednesday and the fourteenth was Thursday. In order to compensate for injustices that might arise in a shortened year, the dates on which payments of various sorts were due were postponed by eleven days. Thus taxes due on the former first day of the year, March 25, were to be payable on April 5 and so they continue to this day.

Franklin notes in his *Almanac* a brief history of the calendar and a summary of the new act, and ends the preface to the document with these statements—

At the Yearly Meeting of the People called *Quakers*, held in *London*, since the Passing of this Act, it was agreed to recommend to their Friends a Conformity thereto, both in omitting the eleven Days of *September* thereby directed to be omitted, and beginning the Year hereafter on the first Day of the Month called *January*, which is henceforth to be by them called and written, *The First Month*, and the rest likewise in their Order, so that *September* will now be the *Ninth Month*, *December* the *Twelfth*.

This *Act of Parliament* as it contains many Matters of Importance, and extends expressly to all the *British Colonies*, I shall for the Satisfaction of the Publick, give at full length: Wishing withal, according to ancient Custom, that this *New Year* (which is indeed a New Year, such an one as we never saw before, and shall never see again) may be a happy Year to all my kind Readers.

The change created a furor in England. A popular song, quoted in P. W. Wilson, *Romance of the Calendar*,[4] was a mixture of militant Protestantism and a resentment of change—

In seventeen hundred and fifty three
The style it was changed to popery
But that it was liked we dont all agree
Which nobody can deny.

Macclesfield's son ran for election to the House of Commons in 1754 but met opposition on the score that his father had robbed the people of eleven days. Hogarth brought this incident into one of his four "election paintings" where a demonstrator carries a placard with the words

[4] P. W. Wilson, *Romance of the Calendar* (New York, W. W. Norton, 1937.) The song consists of three verses of which this is the first.

"Give us back our eleven days." Chesterfield was made president of the Royal Society in 1752 and held that position for twelve years. Bradley was made Astronomer Royal.

The change in the calendar was noted in detail in John Potter's *System of Practical Mathematics*, London, 1753. The title page of this volume lists the items considered from Vulgar and Decimal Fractions to Astronomy and Dialling and then adds—

WITH

A plain Account of the *Gregorian* or New STYLE, settled by Act of Parliament; the Method of finding the Epact, Moon's Age, Tides, &c.

With necessary TABLES:

Particularly the Table calculated by the Right Honourable GEORGE Earl of *Macclesfield*, for finding *Easter*.

Nicholas Pike inserted a section on the Gregorian Calendar in his *Arithmetic for the People of the United States* (Newburyport, 1788). His material is so closely similar to that of Potter as to suggest either direct plagiarism or more likely the use of a common source which might well have been Macclesfield's speech in Parliament which was printed separately and widely distributed. Whatever the source of Pike's material, the subject was timely and important in 1753, but it must have had much less significance in 1788.

So far as the Julian calendar is concerned, it should be noted here that certain churches, among them the Russian Orthodox, the Russian Greek Catholic, and the Ukranian Catholic, keep to the Julian calendar despite the fact that Russia adopted the New Style in 1918, and the Greek Orthodox Church adopted it in the nineteen twenties. The Julian calendar dropped another day behind the Gregorian one in 1800 and still another day in 1900. Accordingly, Christmas Day in the churches noted above, is celebrated thirteen days later than the twenty fifth of December in the civil calendar, or on January 7. The date of Easter sometimes coincides with the Easter of the Gregorian calendar, but may be a month later. On certain districts in Alaska, there are two Christmas holidays—a survival from the Russian days.

References

Beside the titles mentioned in the footnotes, the reader might consult the article on the Calendar in the *Encyclopaedia Britannica*, the articles on George Parker, Philip Dormer Stanhope, and James Bradley in the *Dictionary of National Biography*, and the *New Oxford English Dictionary* for the history of the words Style and Leap year. An important reference in regard to Chesterfield is *Lord Chesterfield and His World* by Samuel Shellabarger, Little, Brown and Co. The material on *Poor Richard's Almanac* is quoted by permission of the Historical Society of Pennsylvania.

Mathematics in Engineering Research

(Continued from page 335)

people automatically to "size up" in a quantitative fashion every situation they encounter. The possession of this habit to some degree is essential to outstanding work in engineering research, or in any phase of life. It should be mastered at an early age, and cannot be mastered unless the processes of elementary mathematics have themselves been mastered by repetitive use until their use is as natural as walking. Teachers of mathematics are in the key position. More power to you!

HAVE YOU SEEN?

In *Scripta Mathematica* for September–December 1951

"The Place of Mathematics in Modern Education" by Carroll V. Newsom

"A Mathematical Reappraisal of the Corpus Platonicum" by Domhnall A. Steele

"Imaginary Elements in Pure Geometry—What They Are and What They Are Not" by Nathan Altshiller Court

"The Golden Age" by Carl B. Boyer

"The Trisection of Horn Angles" by Edward Kasner and Irene Harrison

"The *Liber Abaci* Through the Eyes of Charles S. Pierce" by Carolyn Eisele

The Number Π

By H. von Baravalle
Adelphi College, Garden City, N. Y.

Two outstanding constants of mathematics have been dealt with in previous articles in The Mathematics Teacher, the number e, the base of the natural logarithms[1] and the number G, the ratio of the Golden Section.[2] To complete this series, the present article takes up the third and best known constant, the number π.

As its symbol indicates (π stands for periphery), it represents the ratio of the two outstanding dimensions of the circle, the way around it and the distance across it.

$$\pi = \frac{\text{circumference of a circle}}{\text{diameter of the circle}}.$$

Expressing the diameter in terms of the radius r, we obtain the formula for the circumference of the circle c:

$$\frac{c}{2r} = \pi; \qquad c = 2\pi r.$$

This is by far not the only ratio in which this constant appears. For instance, π is also the ratio of the area of a circle A to the area of the square erected on its radius r:

$$\pi = \frac{A}{r^2}; \qquad A = r^2\pi.$$

It further appears in many other formulae. The volume (V) of a circular cylinder with a base-radius r and altitude h is:

$$V = r^2\pi h$$

and of a circular cone—

$$V = \frac{r^2\pi h}{3}.$$

The surface of a sphere is:

$$A = 4r^2\pi$$

and its volume—

$$V = \frac{4}{3} r^3\pi.$$

The domain of π also extends beyond circular structures. The area of an ellipse with the semi-axes a and b is

$$A = ab\pi$$

and the volume of an ellipsoid with the three semi-axes a, b, and c is

$$V = \frac{4}{3} abc\pi.$$

The area enclosed in a cardioid drawn in Figure 1 as an envelope of circles is

$$A = \frac{3}{2} a^2\pi$$

in which a stands for the diameter of the circle, whose circumference is indicated by the dotted line.[3]

Further examples of curves whose formulae contain π are the roses. The area enclosed by a three-leaved rose (black portions in Figure 2) is

$$A = \tfrac{1}{4}a^2\pi$$

in which a stands for the radius of the circle circumscribed around it. The area enclosed by a four-leaved rose (black area in Figure 3) is

$$A = \tfrac{1}{2}a^2\pi$$

in which a again denotes the radius of the circumscribed circle. The volume of a ring

[1] December 1945 issue (Volume XXXVIII, No. 8).

[2] January 1948 issue (Volume XLI, No. 1).

[3] To construct Figure 1, the dotted circle is divided into thirty-two equal parts. Each of the thirty-two points of division becomes the center of a circle whose radius is its distance from the highest point on the circle (upper end of vertical diameter).

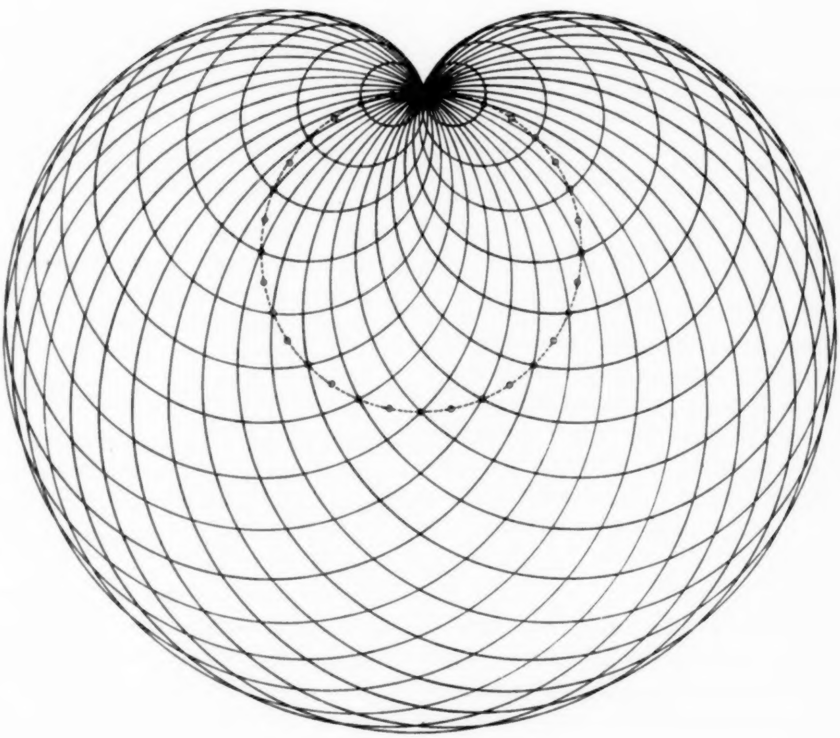

Fig. 1. The cardioid.

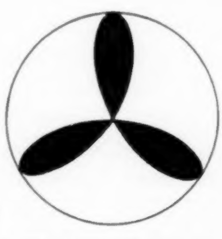

Fig. 2. The three-leaved rose.

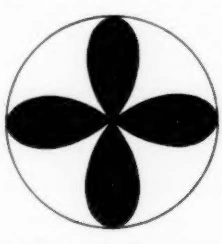

Fig. 3. The four-leaved rose.

(torus), obtained by rotating a circle with radius a about an axis in the same plane at a distance of b units from the center of the circle is expressed in the following formula:

$$V = 2a^2b\pi^2.$$

The volume of the solid of rotation produced by rotating an astroid about one of its axes is:

$$V = \frac{32}{105}a^3\pi.$$

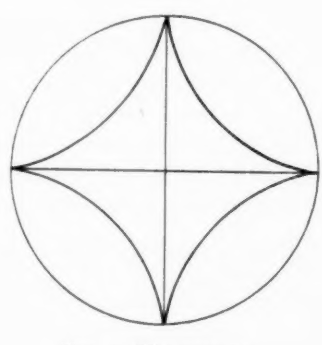

Fig. 4. The astroid.

Here a represents the distance of any of the star points from the center, the radius of the circumscribed circle. The surface area of the same solid of rotation is:

$$A = \frac{12}{5}a^2\pi.$$

The formula for the volume of the solid $x^{2/3}+y^{2/3}+z^{2/3}=a^{2/3}$ whose traces on the coordinate planes are astroids, also contains π:

$$V=\frac{4}{35}\,a^3\pi.$$

All these formulae are obtained by integral calculus.

We can also go beyond areas, surfaces and volumes to find π again in a variety of relationships. A semicircle (radius r), cut out of sheet metal and balanced on a point, will be in equilibrium only when the point of support lies on its axis of symmetry at a distance d from the center, which is:

$$d=\frac{4r}{3\pi}\cdot$$

π even appears in formulae of probability, statistics and in the field of an actuary.

Any vibration, mechanical, acoustical or electrical, proceeds with varying speed. By determining the distance covered by a point on a vibrating musical chord between its extreme positions during a certain time unit, we obtain its average speed of motion. The actual speed of the point is greater every time the chord is near to passing its middle position. It is less than the average speed every time the point finds itself near to one of its extreme elongations. The maximum speed occurs when the point passes in either direction through its position of rest. This maximum speed is in any vibration exactly $\pi/2$ times the average speed.[4] As this holds good for vibrations accompanying every sound of our own vocal chords and in the air around us, π is contained in every word and sentence we say.

The value of π up to 22 decimal places is

3.1415926535897932384626. . .

These successive numerals are the same as the number of letters contained in the successive words of the French verse:

> *"Que j'aime a faire apprendre*
> *Un nombre utile aux sages.*
> *Immortel Archimède, artiste ingénieur,*
> *Oui de ton jugement peut priser la valeur."*

Translation: "How I like to teach a number, useful to the learned: Immortal Archimedes, skillful investigator, yes, the number can tell the praise of your judgment."

Until recently π had been calculated to 707 decimal places. This figure had been obtained by an Englishman, William Shanks, in 1853. With the help of the modern electronic computing machines, the number of decimal places has now been extended to over 2,000.

The history of the number π dates back 3,500 years, as far as historical records show. The Egyptian Rhind Papyrus, dating back as far as 1700 B.C., gives directions for obtaining the area of a circle. Expressed in modern symbols, its formula with A for the circle's area and d for its diameter, is as follows:

$$A=\left(d-\frac{1}{9}\,d\right)^2=d^2\left(1-\frac{1}{9}\right)^2$$

$$=4r^2\left(\frac{8}{9}\right)^2=r^2\,\frac{4.64}{81}=r^2\,\frac{256}{81}\cdot$$

The fraction 256/81, which here takes the

---

[4] The differential equation of a vibration is

$$\frac{d^2x}{dt^2}=-a^2x$$

and its complete solution is $x=r\sin(at-\alpha)$. In this equation, c and a represent arbitrary constants. For $x=0$ at $t=0$, the solution is:

$$x=c\sin at.$$

sin at. Maximum speed:

$$\frac{dx}{dt}=ac\cos at;\quad\text{for } t=0:\quad \left.\frac{dx}{dt}\right|_{\max}=a\cdot c.$$

Average speed:

$$\text{For}\quad at=\frac{\pi}{2};\quad x=c\quad\text{and}\quad \frac{x}{t}=\frac{2ac}{\pi}\cdot$$

The ratio

$$\frac{\left.\frac{dx}{dt}\right|_{\max}}{x}$$

is therefore: $\pi/2$.

place of π, equals in decimals 3.16050 · · ·. Compared with π, (3.14159 · · ·), the difference is 0.01891 · · ·, or less than 1/50.

Archimedes expresses π numerically as follows:

$$3\frac{1}{7} > \pi > 3\frac{10}{71}\,.$$

Expressed in decimals, the same relationships would read:

$$3.142857\cdots > \pi > 3.140845.\cdots$$

Midway between these two values of Archimedes lies the number 3.141851, which, compared with π is only 0.000259 · · · or about $2\frac{1}{2}$ ten-thousandths greater. In ancient China, π was expressed by Ch'ang Höng (125 A.D.) as $\sqrt{10}=3.162\cdots$, the accuracy of which is only slightly less than the value given in the Egyptian papyrus. In 265 A.D. Wang Fan expressed the value of π by the fraction 142/45, or 3.15555 · · ·. In 470 A.D. Ch'ung-chih gave a different fraction: 355/113, or 3.1415929 · · ·, which is correct all the way out to 6 decimal places. In India Aryabhata (510 A.D.) expressed π in this way: "Add 4 to 100, multiply by 8 and add 62,000. This is the approximate circumference of a circle whose diameter is 20,000." Thus π appears as the fraction 62832/20000, which resolves to 3.1416, and is less than one ten-thousandth off.

Though some of these values are sufficiently accurate to have met the practical demands of their times, none reveals any mathematical regularity for the value of π. Against the background of the philosophies of antiquity, one can appreciate the great disappointment which this fact caused to mathematicians and philosophers. This failure regarding the outstanding ratio of the most perfect curve to conform to any pattern of mathematical regularity was considered as a blemish upon the divine world order, and never accepted as the ultimate answer.

The anticipations of antiquity regarding π finally proved justified, but the solution was found only as recently as 360 years ago. The value of π was expressed for the first time in a regular mathematical pattern in 1592 by the great French mathematician, Francois Viète (1540–1603), who found:

$$\pi = 2\cdot\frac{1}{\sqrt{\frac{1}{2}}\cdot\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}\cdot\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}}\cdots}\,.$$

The denominator is an infinite product of expressions of square roots with a regular structure. The possibility of one such development suggests the possibility of other simpler ones; and actually, in 1655, John Wallace (1616–1703), an English mathematician found:

$$\pi = 4\cdot\frac{2\cdot4\cdot4\cdot6\cdot6\cdot8\cdot8\cdot10\cdot10\cdot12\cdot12\cdots}{3\cdot3\cdot5\cdot5\cdot7\cdot7\cdot9\cdot9\cdot11\cdot11\cdot13\cdots}\,.$$

Here π is expressed by infinite products of numbers, this time in both numerator and denominator of a fraction, but without any roots. In the numerator we find the even numbers, in the denominator the odd numbers. Both appear in pairs with the exception of the first factor in the numerator. Only three years later, in 1658, Viscount Brounecker (1620–1684) expressed the value of π as a continued fraction:

$$\pi = 4\cdot\cfrac{1}{1+\cfrac{1^2}{2+\cfrac{3^2}{2+\cfrac{5^2}{2+\cfrac{7^2}{2+\cfrac{9^2}{2+\cdots}}}}}}$$

which again shows complete regularity, the only varying figures being the squares of the odd numbers.

Progress was on the march. The same century brought the final presentation of π as the limit of an infinite series of the simple fractions made up of the odd numbers as their denominators and with alternating signs, the Leibnitz Series. The regularity which was impossible in decimal ex-

pressions of the value of π now became possible through an infinite series of common fractions. Actually, this expression in fractions was more in keeping with the work of the thinkers of antiquity than was that in decimals, which have been in use only since the sixteenth century. That the series is infinite (the transcendence of π was proved by F. Lindemann in 1882) makes the result even more dynamic.

The Leibnitz Series is a fruit of the calculus obtained by one of its inventors. It is derived from expanding the function of arctangent according to Maclaurin's series.

$$f(x)=f(0)+\frac{f'(0)}{1!}\,x+\frac{f''(0)}{2!}\,x^2+\frac{f'''(0)}{3!}\,x^3$$
$$+\cdots+\frac{f^{(n)}(0)}{n!}\,x^n+\cdots.$$

The form for arctan x thus reads:

$$\arctan x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}-\frac{x^{11}}{11}$$
$$+\cdots+(-1)^{n-1}\frac{x^{2n-1}}{2n-1}+\cdots,$$

which converges for all values of x within the limits

$$-1\leqq x\leqq 1.$$

Substituting $x=1$ for an angle of 45° (in radians $45°=\pi/4$; $\tan 45°=1$) we obtain:

$$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}$$
$$-\frac{1}{11}+\frac{1}{13}-\frac{1}{15}+\cdots$$

or

$$\pi=4\cdot\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}\right.$$
$$\left.-\frac{1}{11}+\frac{1}{13}-\frac{1}{15}+\cdots\right).$$

The Leibnitz Series has not been surpassed in all subsequent history in point of its outstanding simplicity. The only later additions were devices for calculating larger numbers of decimals with less effort in the process of computation,—in other words, by finding means of developing π through faster convergencies.

By expanding the arcsine in the same way we obtain the formula:

$$\arcsin x=x+\frac{1}{2}\cdot\frac{1}{3}\,x^3+\frac{1\cdot 3}{2\cdot 4}\cdot\frac{1}{5}\,x^5$$
$$+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{1}{7}\,x^7$$
$$+\frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8}\cdot\frac{1}{9}\,x^9+\cdots$$

which converges for all values of x within the limits of $-1\leqq x\leqq 1$. Substituting $x=1$, we obtain for arcsin 1, corresponding to an angle of 90°, or, in radians, to $\pi/2$, the formula:

$$\frac{\pi}{2}=1+\frac{1}{2}\cdot\frac{1}{3}+\frac{1\cdot 3}{2\cdot 4}\cdot\frac{1}{5}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{1}{7}$$
$$+\frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8}\cdot\frac{1}{9}+\cdots$$

a series which, though more complicated than the Leibnitz Series, converges faster. Further series show a still greater convergence—for instance, that which Abraham Sharp used in 1717 to calculate the value of π to 72 decimal places:

$$\pi=6\cdot\frac{1}{\sqrt{3}}\cdot\left(1-\frac{1}{3\cdot 3}+\frac{1}{3^2\cdot 5}-\frac{1}{3^3\cdot 7}\right.$$
$$\left.+\frac{1}{3^4\cdot 9}-\frac{1}{3^5\cdot 11}+\cdots\right).$$

To find the value of π geometrically, Deinostratus (350 B.C.) used a curve called the Quadratrix. Its construction is shown in Figure 5. Above and below a horizontal base AB, a quarter of a circle with A as its center and AB as its radius is drawn and divided into equal parts. In Figure 5 there are 8 equal parts above and 8 below the base. Then the perpendicular radii are divided into the same numbers of equal parts as the quarters of the circles and through every point of division

a horizontal line is drawn. After also adding a radius through each point of division on the circle, we start with the highest point C and mark the point where the next horizontal line and the next radius intersect, and then continue marking the

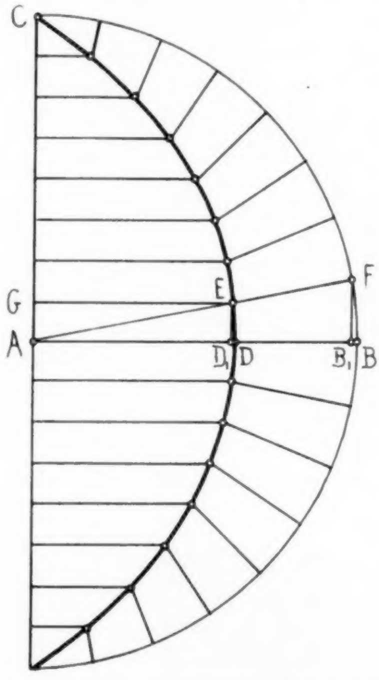

FIG. 5. Geometric construction of the value π.

intersection points of the second horizontal line and the second radius and so forth. The curve passing through these points is the Quadratix. Where it cuts the base AB is the point D and the ratio of the line segments AB and AD is[5]

$$\frac{AB}{AD}=\frac{\pi}{2}\,.$$

The geometric aspects of π lead to the famous problem of the quadrature of the circle, the task of constructing a square (quadratum) whose area equals the area of a given circle. The curve in Figure 5 also derives its name from this problem. An outstanding contribution to the quadrature of the circle was made by Archimedes who found that the area of a circle equals the area of a right triangle, one of whose legs equals the radius and the other the circumference of the circle. This discovery established an equality between the curved area of a circle and the area of a form bounded only by straight lines, and made possible the construction of the quadrature of a circle immediately upon straightening out its circumference. The latter task, so easily performed in actuality every time a wheel rolls over a road imprinting on the road its exact circumference with each revolution, has none the less been an age-long challenge to masters of geometric construction. Its complete solution is possible only by the use of higher curves. Numerous approximations of this geometric construction have been found, however, which for practical purposes represent a solution. Figure 6 shows the approximation constructed by Kochansky. Through the end-points of the vertical diameter are drawn two tangents to the circle. On each of these tangents a certain point is marked. On the lower tangent this point A is three times the length of the radius of the circle away from the point of tangency, while on the upper the point B is fixed at the intersection of the tangent with the prolonged radius drawn at an

[5] The length of the arc BC being one-quarter of the circumference of a circle, is

$$\frac{2r\pi}{4}=r\,\frac{\pi}{2}\,.$$

Its ratio to the radius r is therefore $\pi/2$. The ratio of one-eighth of the arc BC to one-eighth of the radius is therefore also $\pi/2$. The length of the perpendicular from E to AB equals $ED_1=AG$ which is by construction one-eighth of the radius AC; BF is $\frac{1}{8}$ of BC. Therefore, the ratio BF to ED_1 is still $\pi/2$. What holds good for the eighths holds good for any other fraction. The smaller each part of the arc BC becomes, the closer it approaches the length of the perpendicular FB_1. Through the similarity of the triangles $\triangle AB_1F$ and $\triangle AD_1E$ we obtain the proportion:

$$\frac{AB_1}{AD_1}=\frac{FB_1}{ED_1}\,.$$

With an increasing number of points of division and the angle FAB decreasing in size, B_1 approaches B, D_1 approaches D, and the ratio FB_1/ED_1 the ratio FB/ED_1 which equals $\pi/2$.

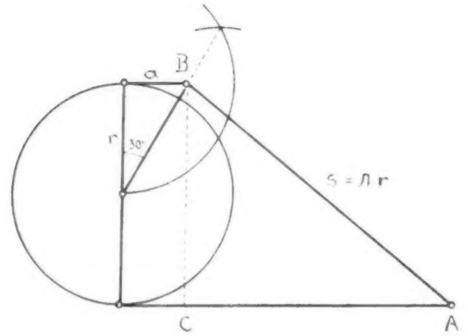

FIG. 6. Construction by Kochansky.

angle of 30° to the vertical diameter. The distance AB is then equal to π times the radius.[6] The approximation provides a difference of less than 0.0001, which lies beyond the graphical limit of precision of Figure 6.

With the help of Kochansky's construction, it is possible to effect the quadrature of the circle, as shown in Figure 7, in two steps. In the diagram to the left we recognize Kochansky's construction. The resulting distance is used as the base of a rectangle with an altitude equal to the radius of the circle. According to Archimedes, the area of the circle equals the area of a triangle whose base is the circumference of the circle and whose altitude is the radius. Therefore, it also equals the rectangle whose base is half the circumference of the circle and whose altitude is the radius. The next step consists in transforming the area of the rectangle into a square—a step accomplished as indicated in Figure 8. The rectangle $ADEF$ is the same as the one in Figure 7. By construction, DB is equal to DE and the intersection of the semicircle above AB with the prolongation of DE determines the point C. $\triangle ABC$ is a right triangle with the altitude h. The area of the square with h as its side equals the area of the rectangle $ADEF$.[7]

The four areas which are marked in black in Figure 7 are equal to one another and show in their sequence from left to

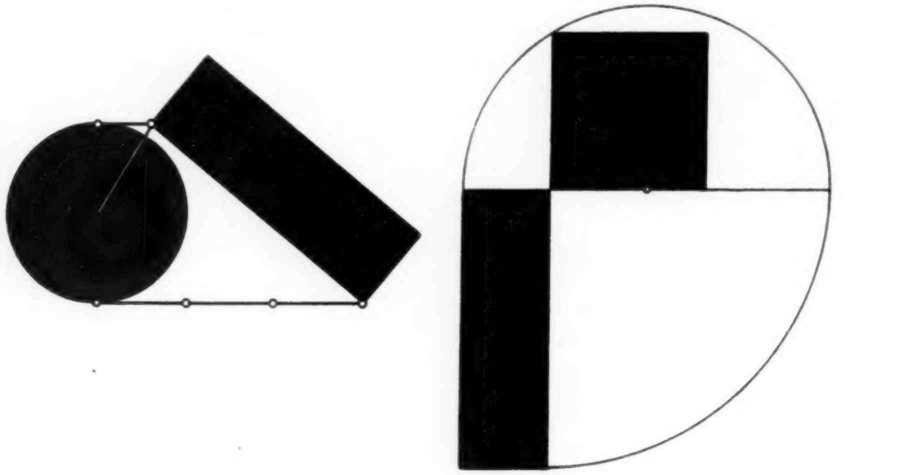

FIG. 7. Construction of the quadrature of a circle.

[6] AB computed as the hypotenuse of the right triangle ABC, with its vertical leg $2r$, and its horizontal leg $3r$ minus the distance a, which is one-half the length of the base of an equilateral triangle with the altitude r $(a=r/\sqrt{3})$ is:

$$AB=\sqrt{(2r)^2+\left(3r-\frac{r}{\sqrt{3}}\right)^2}$$

$$=r\sqrt{4+\frac{(3\sqrt{3}-1)^2}{3}}=r\cdot 3.14153.$$

[7] $AD=\dfrac{h}{\tan\sphericalangle CAD}$;

$DE=DB=h\cdot\tan\ \sphericalangle BCD$; $\sphericalangle BCD=\sphericalangle CAD$ (angles whose sides are perpendicular). Therefore

$$AD\cdot DE=\frac{h}{\tan\sphericalangle CAD}\cdot h\tan\sphericalangle CAD=h^2.$$

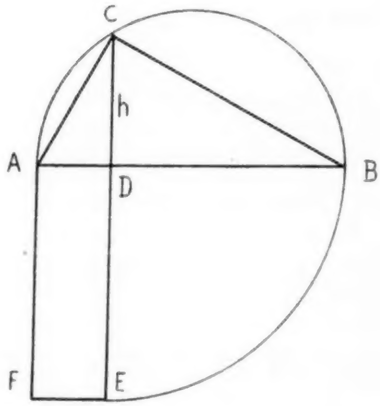

FIG. 8.

right the completion of the quadrature of the circle.

Finally, comparing the three great constants of mathematics, G, e, π:

$$G = 0.6180339887 \cdots$$

$$e = 2.7182818284 \cdots$$

$$\pi = 3.1415926535 \cdots$$

in the form of continued fractions:

$$G = \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}}}}$$

$$e = 1 + 1 + \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cdots}}}}}$$

$$\pi = 4 \cdot \cfrac{1}{1+\cfrac{1^2}{2+\cfrac{3^2}{2+\cfrac{5^2}{2+\cfrac{7^2}{2+\cdots}}}}}.$$

G expresses itself through repetitions of the number 1 only, and e through repetitions of 1 and the powers of 2. In the fraction of π, the variable element is the series of the squares of the odd numbers.

There is an approximation between π and G which played a major role in the history of the investigations of the proportions of the Great Pyramid in Egypt:

$$\left(\frac{\pi}{4}\right)^2 = 0.6168 \cdots$$

$$G = 0.6180 \cdots.$$

Though the difference between the two values is only 0.0012, their closeness is merely incidental and has no basis in mathematical law. Neither is there any mathematical connection between G and e.

Different is the case with the constants π and e. Between them there is a distinct mathematical relationship. The expansion of e^x, according to MacLaurin's Series is

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots + \frac{x^n}{n!} + \cdots$$

and that of $\sin x$ and that of $\cos x$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \cdots + (-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \cdots + (-1)^{n-1}\frac{x^{2n-2}}{(2n-2)!} + \cdots.$$

The two series for $\sin x$ and $\cos x$ together furnish all the terms of the series of e^x, with the only discrepancy in their signs, a difficulty which does not exist for the hyperbolic sines and cosines. Their expansions have only positive terms:

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

$$+\frac{x^{2n-1}}{(2n-1)!}$$

$$\cosh x = 1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots$$

$$+\frac{x^{2n-2}}{(2n-2)!}\cdot$$

Therefore it readily appears that $e^x = \sinh x + \cosh x$. An analogous result for the trigonometric functions can be obtained if we substitute for x its product with the imaginary unit:

$$e^{ix} = 1+ix-\frac{x^2}{2!}-i\frac{x^3}{3!}+\frac{x^4}{4!}+i\frac{x^5}{5!}$$

$$-\frac{x^6}{6!}-i\frac{x^7}{7!}+\cdots$$

and separate the real and imaginary terms. Thus, the result is:

$$e^{ix} = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$$

$$+i\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots\right)$$

or

$$e^{ix} = \cos x + i \sin x,$$

a formula which finds wide application in the solving of differential equations, particularly of those connected with all types of vibrations. Substituting $x=\pi$, we obtain:

$$e^{i\pi} = \cos \pi + i \sin \pi = -1 + i\cdot 0 = -1$$

which results in the formula,

$$e^{\pi i} = -1.$$

It is this formula which prompted David Eugene Smith to use it in the mathematical credo placed in his library:

The Science Venerable:

Voltaire once remarked—"One merit of poetry few will deny; it says more and in fewer words than prose." With equal significance we may say, "One merit of mathematics few will deny; it says more and in fewer words than any other science." The formula, $e^{\pi i} = -1$ expresses a world of thought, of truth, of poetry and of religious spirit, for "God eternally geometrizes."

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HAVE YOU SEEN?

In *The Mathematical Gazette,* February 1952

"A Visual Aid Technique for Area Formulae" by R. H. Collins

"Three Geometrical Mechanisms: 1) A Device for solving $a \sin x - b \cos x = 1$; 2) A Linkage for the Mechanical Construction of Regular n-gons; 3) A Linkage for Dividing an Angle Mechanically into any Number of Equal Parts" by G. D. C. Stokes

"Geometry of Many Dimensions" by Norman H. Smith

In *Scientific American,* April 1952

"The Strange Life of Charles Babbage" by Philip and Emily Morrison. "This remarkable Englishman tried to build modern computing machines a century ahead of their time. . . . His monument, by no means wholly beautiful but very grand, is the kind of research that is epitomized today by the big digital computers."

"Natural Selection in Language" by Joshua Whatmough. "Some recent studies making use of history, grammar and the mathematical theory of communication shed light on the origin of language types and even of language itself. . . . Now that language has finally been reduced to mathematical formulation, we shall understand it better."

Speakers' Bureau

By Mary C. Rogers

Roosevelt Junior High School, Westfield, New Jersey

Early in May 1950, Mr. H. W. Charlesworth, President of the National Council of Teacher of Mathematics, instructed that initial steps be taken toward organizing a Speakers' Bureau service for Affiliated Groups. This action was taken as an outgrowth of a unanimous request for such service expressed by the First Delegate Assembly at Chicago in April 1950.

We are happy to make this, our first report of progress to the members of the National Council. It should be considered as a description of *preliminary* plans and procedures—*initial stages* in a long-term program of service which if successful should become increasingly functional over the years.

From the very beginning this problem has given considerable study. A tentative report of possible procedures was submitted to the Board of Directors of the National Council at its annual meeting in Pittsburgh in March 1951. This report, qualified by certain amendments, was then presented to the Second Delegate Assembly and their reactions reported back again to the Board. After more intensive study, a later report was made to the Board at the St. Olaf convention in August 1951. It was here suggested that the Speakers' Bureau service be tried out regionally before any attempt be made to extend the service on a nation-wide basis. The Bureau was instructed to set up plans for trial service in the New York metropolitan area. The initial trial Speakers' Bureau service in this area would be evaluated for its effectiveness. If this service proved to be reasonably functional, similar regional services would then be organized in other areas throughout the country. Eventually, there might be some exchange of services between regional areas.

An Advisory Committee was appointed to amend and supplement existing plans; to formulate policies; and to recommend specific techniques to be followed. The following people constitute this committee:

Mary C. Rogers, Roosevelt Junior High School, Westfield, N. J., Chairman.
Howard F. Fehr, Teachers College, Columbia University, New York City.
M. Albert Linton, President, Philadelphia Council of Teachers of Mathematics, Penn Charter School, Philadelphia, Pa.
Alice M. Reeve, President, Association of Mathematics Teachers of New York State, Senior High School, Rockville Centre, N. Y.
Hubert B. Risinger, Past-President, Association of Mathematics Teachers of New Jersey, Davey Junior High School, East Orange, N. J.

Leaders from the various states and sections in the New York metropolitan area were interviewed informally at St. Olaf for their reactions to a regional speakers' bureau service. They expressed interest in the program and offered their support, if the program were begun modestly and guided in growth toward a long-term service which should gain impetus and strength as its initial successes become apparent.

In consideration of these developments, the Advisory Committee of the Speakers' Bureau drew up the following specific recommendations. These recommendations were then approved by the Executive Committee of the National Council Board and authorization given for their immediate activation.

1. Contact the leaders already approached in the various states and sections of the New York metropolitan area, review the objectives already presented to them, and solicit their frank comments and constructive criticisms as well as their active cooperation.
2. Request that a local Speakers' Bureau chairman be appointed in each

of these areas; that he, with the endorsement and advice of the Executive Board of his local association, organize a committee to work with him in surveying his area for speaking talent.

3. The details of procedure to be followed in canvassing local areas for speaking talent should be determined by the local chairman, his committee, and their local sponsoring association. Specific procedure should be suited to the type of area serviced.
4. At the end of the school year, 1951–52, the local committees should report their findings to the Central Speakers' Bureau. This report should include:
   a. A suggested list of speaking talent, comprised of
      1) speakers who have already received state and national recognition.
      2) newly discovered talent, persons who have been recommended to the committee through authoritative sources.
   b. The specific topic or topics in which these speakers have their strength and have shown ability for effective communication to an audience.
5. Upon receipt of the reports from the local committees, the Central Speakers' Bureau will study the lists submitted for the possibility of creating a source of speakers under the auspices of the National Council of Teachers of Mathematics.
6. The availability of these services will be made known through THE MATHEMATICS TEACHER, the Newsletter of the Affiliated Groups, and through the local publications of interested associations. Associations wishing to use this service, should then contact the Central Speakers' Bureau for complete listings and for certain detailed information which they may desire.
7. If these plans are activated promptly, it is believed that Speakers' Bureau services—the securing of speakers through the Bureau—will be possible early in the school year, 1952–53.

Early in the Fall of 1951, the Council of the Association of Teachers of Mathematics in New England established a local Speakers' Bureau consisting of a corps of members, each serving for one year. Professor William R. Ransom, Reading, Massachusetts was appointed chairman of this local bureau. A list of speakers was assembled and their names made available to mathematics departments, clubs and other mathematical organizations within New England. An invitation was extended to all interested groups to make use of the facilities so offered. In addition, membership of the Association was circularized individually with a full description of Bureau plans and services. Interest in the Regional Speakers' Bureau has been expressed and a willingness shown to cooperate with the National Council program.

Dr. Catherine A. V. Lyons, President of the Pennsylvania Council of Teachers of Mathematics, reports keen interest among Pennsylvania leaders in the proposed Regional Speakers' Bureau program. They see great possibilities in the services it can render and are eager to give it their support. Initial plans of the Pennsylvania Speakers' Bureau Committee were presented at the annual meeting of the Council held at the State Teachers College, Indiana, Pennsylvania on March 29, 1952, and action was taken for their implementation.

Word comes to us from within New York State that steps are being taken toward the ultimate endorsement and active support of the Regional Speakers' Bureau. Leaders of strong county and city groups have been approached for their reactions and advice. An article appeared in the February Newsletter of the New York State Association urging the formation of a local Speakers' Bureau. The

annual meeting of this Association was held at Syracuse University on Saturday, May 3, 1952 and, at that time, plans for the New York State Speakers' Bureau were presented to the membership.

The Regional Speakers' Bureau has been enthusiastically endorsed in the State of Maryland. A local committee has been set up under the leadership of Miss Margaret Heinzerling, Chairman of the Mathematics Section of the Maryland State Teachers Association and Miss Elizabeth Gardner of the Robert E. Lee Junior High School in Baltimore. Plans for a state-wide canvass of speaking talent are taking shape. It is reasonably certain that the Maryland survey will be completed within the next four months.

The Association of Mathematics Teachers of New Jersey has also achieved much in its support of the Regional Speakers' Bureau service. A committee has been appointed by the New Jersey Council and is under the leadership of Dr. Hubert B. Risinger of East Orange, New Jersey. This committee has submitted very definite plans of procedure which have been approved by the Council, and a "go-ahead" signal has been given. As its initial active step, the committee has circularized Association members individually, outlining further plans of action—soliciting the support and advice of all persons concerned. The One Hundredth Regular Meeting of the Association was held at Newark, New Jersey, on Saturday, March 1, at which the direct reactions of the membership personnel were solicited. This committee now plans to begin its State-wide canvass of speaking talent. The committee is quite optimistic that this survey will be completed by June 1, 1952, and a report ready for the Central Speakers' Bureau soon thereafter.

The Advisory Committee of the Regional Speakers' Bureau is very much interested in your reactions to this first report. Do you favor what we have already accomplished and are planning to do? Are there improvements in procedure which you would suggest? Would you supplement anticipated services to any great extent in the immediate future? We shall welcome your advice and constructive criticisms for we want our service to you to be of the very highest order.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Membership Record

Mary C. Rogers, *Roosevelt Junior High School, Westfield, New Jersey*

Additional 100% Schools—as of March 1, 1952

1. Sebring, Florida .... Sebring High School
2. Atlantic, Iowa .... Atlantic High School
3. Mason City, Iowa .... Monroe Junior High School
4. Mason City, Iowa .... Roosevelt Junior High School
5. Mason City, Iowa .... Senior High School
6. Plainfield, Illinois .... Plainfield High School
7. Grinnell, Kansas .... Grinnell Rural High School
8. Syracuse, Kansas .... Syracuse High School
9. Wichita, Kansas .... Wichita High School, East
10. Baltimore, Maryland .... Eastern High School
11. West Point, Mississippi .... Mary Holmes Junior College
12. Eureka, Montana .... Lincoln County High School
13. North Platte, Nebraska .... Senior High School
14. Indiana, Pennsylvania .... State Teachers College

Additional "All but One" Schools—as of March 1, 1952

1. Springfield, Illinois .... Springfield High School
2. Penns Grove, New Jersey .... Regional High School

RESEARCH IN MATHEMATICS EDUCATION

Edited by John J. Kinsella
School of Education, New York University, New York 3, N. Y.

The Questions: What should constitute the subject matter of variation and functionality for secondary school students? How can this content be used in teaching problem solving and in correlating mathematics and science instruction?

The Study: Rich, Barnett. *Variation, Its Extension and Application to Problem Solving.* Ph.D. dissertation. Teachers College, Columbia University. 1949.

A survey of the history of mathematics education since 1900 reveals periods in which the teaching of the function concept and the correlating of mathematics and science were the dominant concerns. Since 1940, however, when two significant reports (1) of national stature assigned the function concept a less important role, these two problems have received less attention. This tendency was also apparent in another influential study appearing in 1945 (3). In fact, it is even difficult to find in the 1951 Mathematics Teacher one article that has "function concept" in its title. Although applications of mathematics were emphasized during the war and since, it has not been apparent that the impetus for this trend was a desire to implement a theory of correlation for science and mathematics.

Dr. Rich proposed that one of the major reasons for the neglect of functionality and correlation of mathematics and science might be the failure "to determine what should constitute the proper subject matter of functionality for secondary school students." The heart of his study was to delineate what this subject matter should be. In addition, ways in which this content could be used in teaching problem solving and in correlating mathematics and science were described. What follows is merely an attempt to summarize the most challenging proposals.

1. Instead of beginning a study of functionality with two or more variables, begin with the relationships that exist between *any two values of the same variable.* Three kinds of relationships or comparisons between the two values (x_1, x_2) would be made, namely, the amount of change $(x_2 - x_1 = \Delta x)$, the ratio of the two values $(x_2/x_1 = \not(x))$ and the rate of change $\dfrac{x_2 - x_1}{x_1} = r_x$. It follows that $\Delta x/x_1 = r_x$ and $\not(x) = 1 + r_x$.

2. Use the new symbol, $\not(x)$, defined above. Reasons given for its use are economy in symbolism, elimination of the antiquated symbol for "vary as," avoidance of complex fractions, increased power of generalization and elimination of the unit of measurement in the computations involved in mensurational problems.

3. Concentrate the study of functionality on the three types of the three-variable, basic equations, namely, $xy = z$ and $x + y = z$ and $x^y = z$, through the medium of two-set equations. The three are basic because they involve the six basic operations of elementary algebra and arithmetic. Two-set equations are illustrated in (1) above and are so-called because two sets of values of the variables must be used in dealing with them. The three basic equations are one-set equations because they require only one set of values for their interpretation. From them can be derived the basic two-set equations. From $xy = z$ comes $\not(x)\not(y) = \not(z)$, from $x + y = z$ comes $\Delta x + \Delta y = \Delta z$ and from $x^y = z$ comes $x^{\Delta y}$

$=\rlap{-}(z)$ if x is constant, $\rlap{-}(x)^y=\rlap{-}(z)$ if y is constant and $(x)=z^{\Delta y}$ if z is constant. If there are no constants in $x^y=z$, then $\rlap{-}(y)\rlap{-}(\log\ x)=\rlap{-}(\log\ z)$. As a simple illustration of $xy=z$, one-set cost formula, $C=NP$ with N constant becomes $\rlap{-}(C)=\rlap{-}(P)$ or $\Delta C=N\Delta P$ or $r_c=r_p$. The corresponding interpretations would be that if C be multiplied by K so will P; if P increases by K, C will increase by NK; if the rate of price increase is K so is the rate of cost increase.

4. Through $xy=z$ and $\rlap{-}(x)\rlap{-}(y)=\rlap{-}(z)$ deal with joint variation first, with direct and inverse variation as special cases occurring respectively when a factor or a product is constant. Treat $x+y=z$ first as illustrating combined sense change, direct sense change next resulting from a constant addend and then opposite sense change stemming from a constant sum, z. Treat similarly power variation expressed through $x^y=z$.

5. Organize problems and science relationships around the three types of one-set and two-set equations rather than by the traditional, situated types. For example, $xy=z$ is the key relationship in problems of motion, area, mixture (%), proportion, the lever, work, interest, the general gas law, pressure, the five simple machines, gravitation and Ohms Law in electricity. Thus, the student is encouraged to seek for general relationships that apply to families of problems. This practice should be helpful in the transfer "problem." Also, he comes to see mathematics as an essential instrument in analyzing science problems involving variables and constants. If the habit of seeking relationships rather than memorizing types can be developed in problem solving and the attitude of expecting mathematics to be essential in understanding and analyzing science situations can be engendered through Dr. Rich's program, his study will be of great significance in mathematics education.

6. Extend the above ideas to geometric relationships, e.g., sum of angles of polygons, complementary and supplementary angles, angle measurement, similarity and areas.

7. Extend the use of tables and graphs in studying functionality. Use the composite table-graphs in which numbers appear at the vertices of the grid's squares. Use a greater variety of graphic scales, namely, rate scales and logarithmic scales in addition to the constant difference scales.

This department realizes that space limitations prevent us from making a more complete summary and recommends that Dr. Rich's study, now in book form (2), be consulted for further details and clarification.

References

1. a) Progressive Education Association. *Mathematics in General Education* (Report of the Mathematics Committee of the Commission on Secondary School Curriculum). New York: D. Appleton-Century, 1940. pp. 70–71.
   b) *The Place of Mathematics in Secondary Education* (15th Yearbook of the National Council of Teachers of Mathematics). New York: Bureau of Publications, Teachers College, Columbia University, 1940. pp. 41–42.
2. Rich, Barnett. *Variation, Its Extension and Application to Problem-Solving*. Cambridge: Eagle Enterprises, 1951.
3. "The Second Report of the Commission on Post-War Plans." The Mathematics Teacher, XXXVIII (May 1945).

Recognizing today's urgent need for scientists, engineers, and laboratory technicians of all sorts, the National Association of Manufacturers has published a 32-page booklet, Your Opportunities in Science, which is available in quantities to high school and college students without charge. Teachers of high school mathematics should send for a supply for their classes. The pamphlet shows the young reader that the scientist and technical worker are modern frontiersmen. It demonstrates that one doesn't have to be a genius to qualify—that opportunities are plentiful for boys and girls with many different types of aptitudes, interests and educational levels. It also emphasizes that the qualities needed can generally be developed. Orders should be sent to Special Services Department, National Association of Manufacturers, 14 West 49th Street, New York 20, New York. Be sure to mention The Mathematics Teacher.

MATHEMATICAL RECREATIONS

Edited by Aaron Bakst
135-12 77th Avenue, Flushing 67, N. Y.

This department projected a series of illustrative recreations which might be employed in a mathematics classroom in connection with the subject matter regularly taught throughout the school year. On several occasions this department indicated that a mathematical recreation need not be represented by a drastic departure from the specific topics which are taught. On the contrary, it is the opinion of this department that a mathematical recreation should be incidental to the instructional processes. Furthermore, a mathematical recreation may be confined to a short pastime, or it may encompass an entire topic. True, this may mean a departure from the textbook material, and such a departure may be frowned upon by the tradition-bound teacher. Nevertheless, a departure from textual material may often be a very wholesome practice.

The material presented here belongs to the level of ninth year algebra and it should be considered in connection with the topic on *ratio and proportion*.[1] The objectives (which are usually extremely broad and general) of the teaching of algebra state that the pupils should be encouraged to collect data, to arrange them, to treat them, and so on. Generally, these objectives, however lofty their statements may sound, remain objectives which, at the end of the year remain unattained. More often than not, the textbook is the path of the least resistance. There is yet an algebra textbook to be written which will blaze the path to the pupils' activities which would offer the opportunities to experience the above objective. These activities represent life situations, and if a teacher expects to find them in a textbook, this department recommends a brief reference in St. Luke, 24:5.

The problem of the teaching *ratio and proportion* is not difficult from the pedagogical point of view. But when illustrative materials are needed for these topics, the teacher usually (and suddenly) discovers that there is an unusual want of materials which are of real interest to the pupils. True, we may have plans of floors, geographical maps, pictures of animals. These are the representative "run of the mill materials" which should be classified with all other "busy work" in mathematical studies.

There is one field which is closely tied up with *ratio and proportion* and which is not only practical, but widely used these days. Business, industry, public opinion polls, surveys, and so on, employ a technique which can be easily practiced by the pupils in collecting data. The technical term for this topic is known as the Theory of Attributes.[2] Generally, this theory does not require (in the early stages)more than the algebra of the ninth year level. Conceptually, the processes associated with this theory require the application of common sense. From the point of view of the utilization of the fundamental arithmetic operations (and partially algebraic) this field offers unlimited opportunities for all sorts of computational activities.

[1] This material is based on a portion of the chapter "The Ananias Club" from the forthcoming book *Man, Number and Space* by the editor of this department. All rights reserved.

[2] A detailed and thorough treatment of the Theory may be found in G. Udny Yule, *An Introduction to the Theory of Statistics* (London: Charles Griffin & Co., Ltd., 1937), pp. 11-81.

Let us consider a practical example. This situation may be offered to the pupils as a challenge as well as an activity in collecting data. Suppose that a survey of homes with radio sets and with television sets was made and the following data was secured:

Homes with radio sets, but no television sets	362
Homes with television sets, but no radio sets	227
Homes with radio and television sets	529
Homes with no radio and no television sets	93

Such a survey could have been made by a local store or by a manufacturer for the purpose of determination whether the ownership of a radio set may influence the purchase of a television set. A similar survey may be suggested to the pupils for the same purpose.

The data stated above offer sufficient material for the purposes of the development of the topics on *ratio and proportion*. We shall postpone the analysis of this data, and we shall introduce the symbolism which is employed in connection with the study of *attributes*. This symbolism is algebraic in nature, and this offers an opportunity to develop further the principles of algebraic symbolism.

Let us denote the possession of an attribute by capital letters $A, B, C, \cdots$. The nonpossession of an attribute is denoted by small letters $a, b, c, \cdots$. The number of objects which possess a given attribute is denoted by the symbols (A), (B), (C), $\cdots$. The number of objects which do not possess a given attribute is denoted by the symbols (a), (b), (c), $\cdots$. Then the total population of a survey is denoted as $N=(A)+(a)$, or $N=(B)+(b)$, $N=(C)+(c)$, $\cdots$.

If we have objects which possess two attributes, then their number is denoted by the symbol (AB), (AC), (BC), $\cdots$. If we have objects which possess three attributes, then their number is denoted by the symbol (ABC). The symbol (Ab) represents the number of objects which possess the attribute A but do not possess the attribute B. This symbolism may be extended to any number of attributes.

Note then that $(A)=(AB)+(Ab)$ and $(B)=(AB)+(aB)$.

The data listed above may then be represented symbolically as follows:

Homes with radio sets, but no television sets	(Ab)
Homes with television sets, but no radio sets	(aB)
Homes with radio and television sets	(AB)
Homes with no radio and no television sets	(ab)

$$\text{Then } N=(AB)+(Ab)+(aB)+(ab)$$

If another attribute were included, as, for example, washing machines, then this attribute would be assigned the symbol C, and we would have the following combinations: (ABC), (ABc), (AbC), (Abc), (aBC), (aBc), (abC), (abc). These combinations may be translated into spoken or written language. For example, (aBc) represents the number of homes with no radio sets and no washing machines, but with television sets. This translation should offer the pupils an opportunity in practicing with meaningful algebraic symbolism. With proper guidance they can create their own symbols. An exercise with four symbols A, B, C, and D should not be unduly difficult.

Furthermore, the above list of symbols leads to the formulations

$$(AB)=(ABC)+(ABc),$$
$$(A)=(ABC)+(ABc)+(AbC)+(Abc).$$

The pupils may be challenged to obtain the expressions for (B), (C), (AC), (BC), (a), (b), (c), (ab), (ac), and (bc). For example,

$$(bc)=(Abc)+(abc),$$
$$(c)=(ABc)+(Abc)+(aBc)+(abc).$$

The cue for all these formulations is simple. Note the symbols with the required letters and obtain their sum. The amount of numerical work which may be secured from a table of such data is quite extensive.

The symbols $A, B, C, \cdots, a, b, c, \cdots$ may also represent ratios. Then $A \cdot N =$

(A) and $a \cdot N=(a)$. From this we have that $N=(A)+(a)=(A+a)N$, or $A+a=1$. This interpretation of the symbols A and a may be employed in the usual algebraic sense. Thus, for example,

$$(aB)=aB \cdot N=(1-A)B \cdot N$$
$$=N(B-AB)=(B)-(AB).$$

It should be noted that all the numerical values of the symbols as well as the numerical values of their various combinations must be always positive. If, at any time a negative result is obtained, then the data contains some error which indicates some inconsistency.

The fact that all the numerical values of the symbols (A), (B), $\cdots$, (a), (b), $\cdots$, as well as of their combinations must be all positive offers an opportunity to develop further the meaning of negative numbers.

We may now pose the question whether there is some relation between the ownership of a radio set and of a television set. In other words, would it be reasonable to assume that it is easier to sell a television set to a person who owns a radio set than to a person who does not own a radio set? If there were no relation, then the ratio of the number of television sets and radio sets owners to the number of radio sets owners would be the same as the ratio of the number of television sets owners to the number of no radio sets owners. In other words, if the ownership of a radio set is denoted by the symbol A, and the ownership of television set is denoted by the symbol B, we have the following proportion

$$\frac{(AB)}{(A)}=\frac{(aB)}{(a)}.$$

In a similar manner we can formulate the proportions

$$\frac{(AB)}{(B)}=\frac{(Ab)}{(b)}.$$

From the proportion $a:b=c:d$, we may obtain other proportions, such as $(a+b):b=(c+d):d$ and so on. Employing the same algebraic procedures we may obtain the proportion

$$\frac{(AB)}{(B)}=\frac{(A)}{N}.$$

From the above table of numerical data we have that: $(AB)=529$, $(A)=362+529=891$, $(B)=227+529=756$, and $N=1211$. Then,

$$\frac{529}{756}\neq\frac{891}{1211},$$

because $(529 \cdot 1211)<(756 \cdot 891)$. In other words, there is some relationship between the ownership of a television set and of a radio set.

The above illustrations are not offered as a panacea for the topic on *ratio and proportion*. However, this may be considered as an example of what could be done in an algebra classroom in order to motivate the topic with procedures which are different from the traditional subject matter. This type of treatment would also represent a diversion from the textbook, and this is at times a "recreation" for pupils.

The treatment of *ratio and proportion*, as indicated above, may be correlated with the social studies insofar as this method is applicable to public polls. In other words, this method offers an opportunity for developing a topic along the specific lines which are employed in various fields.

This department would be very much interested in the reactions of the readers of THE MATHEMATICS TEACHER concerning the suggestions which have been made relative to the development and teaching of mathematical recreations. It is the belief of this department that mathematical recreations can be made an integral part of the teaching program. An urgent invitation is issued here for contributions. These will be gratefully accepted.

RELATIONSHIPS FOR RADII OF CIRCLES ASSOCIATED WITH THE TRIANGLE

By Herta Taussig Freitag
Hollins College, Va.

In the April 1951 issue (p. 270), the department of Mathematical Recreations requested problems concerning the relationships of certain radii within a given triangle. The following have proved of interest.

I. Given a right triangle ABC with γ as the right angle. Draw the altitude CD.

A. The sum of the radii of the incircles of triangles CDB, ADC, ABC, respectively, is equal to the length of the altitude CD.

Given:

1. $\triangle ABC$ with $\angle\gamma=90°$
2. $CD\perp AB$; $CD=h_c$
3. Incircle of $\triangle CBD$: center M_1, radius r_1
   Incircle of $\triangle ADC$: center M_2, radius r_2
   Incircle of $\triangle ABC$: center M_3, radius r_3

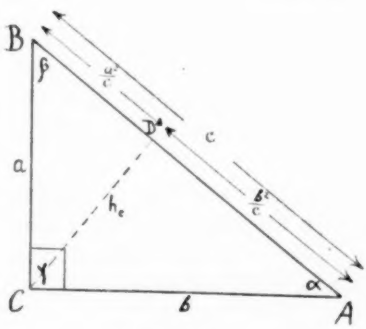

Fig. 1

4. Circumcircle of $\triangle CDB$: radius R_1
   Circumcircle of $\triangle ADC$: radius R_2
   Circumcircle of $\triangle ABC$: radius R_3
5. $S=$area ABC
6. $p=$perimeter ABC

Prove: $r_1+r_2+r_3=h_c$

1. **Proof by means of plane geometry:**

$\triangle CDB\sim\triangle ADC\sim\triangle ABC.$

$$\therefore\ \frac{r_1}{r_3}=\frac{a}{c},$$

or:
$$r_1=\frac{a}{c}r_3.$$

Similarly:
$$r_2=\frac{b}{c}r_3.$$

Hence:
$$r_1+r_2+r_3=\frac{a+b+c}{c}r_3.$$

But
$$r=\frac{2S}{p}.$$

$$\therefore\ r_3=\frac{ch_c}{a+b+c},$$

(1) and $r_1+r_2+r_3=h_c.$

An alternate proof, using concepts of plane geometry, can also be found.

There are several proofs using trigonometric relationships.

2. **Proof by means of analytic geometry where $m=$slope and $e=$equation:**

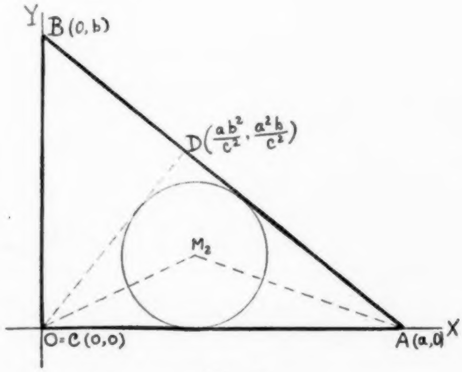

Fig. 2

$$\overline{CD}=\frac{ab}{c}=h_c$$

$$m(CD)=\frac{a}{b}; \qquad m(AB)=-\frac{b}{a}$$

$$m(CM_2)=\frac{a}{b+c}=\frac{c-b}{a}$$

$$m(AM_2)=-\frac{b}{a+c}=\frac{a-c}{b}$$

$$\left.\begin{aligned} &c(CM_2), \quad y=\frac{c-b}{a}\,x \\ &c(AM_2), \quad y=\frac{a-c}{b}\,(x-a) \end{aligned}\right|$$

Let $M_1(x_1, y_1)$; $M_2(x_2, y_2)$; $M_3(x_3, y_3)$. Then:

$$y_2=r_2=\frac{a(a-c)(c-b)}{c(c-a-b)};$$

and by cyclic permutation:

$$x_1=r_1=\frac{b(b-c)(c-a)}{c(c-a-b)}$$

$$\left.\begin{aligned} &c(CM_3), \quad y=x \\ &c(AM_3), \quad y=\frac{a-c}{b}\,(x-a) \end{aligned}\right|$$

$$\therefore \quad x_3=y_3=r_3=\frac{a(a-c)}{a-c-b}$$

and
$$r_1+r_2+r_3=(a-c)\left[\frac{(c-b)(a+b)}{c(c-a-b)}-\frac{a}{c-a+b}\right]$$

$$=\frac{b}{2ac}\,(c^2+a^2-b^2)$$

$$=\frac{ab}{c},$$

or
$$r_1+r_2+r_3=h_c.$$

B. Further relationships suggest themselves:

1. The sum of the radii R_1, R_2, and R_3 of the circumcircles of triangles CDB, ADC, and ABC, respectively, is equal to half the perimeter of triangle ABC.
2. The sum of the areas of the circles inscribed in triangles CDB and ADC, respectively, is equal to the area of the circle inscribed in triangle ABC.
3. The sum of the areas of the circles circumscribed about triangles CDB and ADC, respectively, is equal to the area of the circle circumscribed about triangle ABC.

The respective proofs of these relationships are as follows:

$$R_1=\frac{a}{2}, \qquad R_2=\frac{b}{2}, \qquad R_3=\frac{c}{2}$$

and

$$\left.\begin{aligned} r_1^2&=\frac{a^2}{(a+b+c)^2}\,h_c^2 \\ r_2^2&=\frac{b^2}{(a+b+c)^2}\,h_c^2 \\ r_3^2&=\frac{c^2}{(a+b+c)^2}\,h_c^2 \end{aligned}\right\} \text{(see IA1).}$$

Therefore:

$$(2) \qquad R_1+R_2+R_3=\frac{p}{2}$$

$$(3) \qquad \pi r_1^2+\pi r_2^2=\pi r_3^2$$

$$(4) \qquad \pi R_1^2+\pi R_2^2=\pi R_3^2.$$

IIA. Given *any* triangle ABC. Draw the altitude CD. It is required to develop relationship(s) for the sum of the radii of the incircles of triangles CDB, ADC, ABC, respectively.

If the same symbolism be employed as in IA with the omission of the condition that $\angle\gamma=90°$, the following may be obtained by trigonometric means:

$$(5) \qquad r_1+r_2+r_3=h_c+\frac{2s}{p}\left(1-\cot\frac{\gamma}{2}\right)$$

$$(6)^* \quad r_1+r_2+r_3\cot\frac{\gamma}{2}=h_c.$$

* It may be interesting to note that the relationship

$$(6a) \qquad R_1+R_2+R_3\sin\gamma=\frac{p}{2}$$

holds for the radii of the circumscribed circles.

B. If $\triangle ABC$ is a right triangle, formula (6) reduces to (1).

C. If $\triangle ABC$ is equilateral,

$$a=b=c, \quad p=3a, \quad \angle\gamma=60°,$$

$$r_3=\frac{h}{3}=\frac{a}{2\sqrt{3}},$$

and formula (6) may be reduced to several variations of

$$r_1+r_2+\frac{a}{2}=h.$$

IIIA. Given *any* triangle ABC. Draw the *three* altitudes. Let the orthocenter be O. Relationship(s) between the radii of the circles inscribed in triangles COB, AOC, BOA, respectively, and the radius of the circle inscribed in triangle ABC may be found as follows.

Let:

r_1 = radius of incircle of $\triangle COB$
r_2 = radius of incircle of $\triangle AOC$
r_3 = radius of incircle of $\triangle BOA$
r = radius of incircle of $\triangle ABC$
R_1 = radius of circumcircle of $\triangle COB$
R_2 = radius of circumcircle of $\triangle AOC$
R_3 = radius of circumcircle of $\triangle BOA$
R = radius of circumcircle of $\triangle ABC$

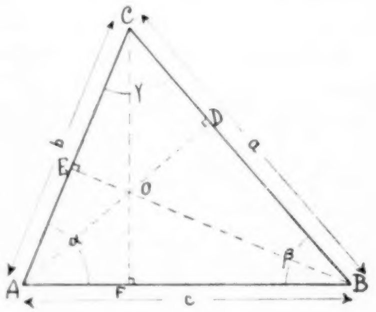

FIG. 3

S_1 = area COB $\quad \dfrac{p_1}{2}$ = semiperimeter COB

S_2 = area AOC $\quad \dfrac{p_2}{2}$ = semiperimeter AOC

S_3 = area BOA $\quad \dfrac{p_3}{2}$ = semiperimeter BOA

S = area ABC $\quad \dfrac{p}{2}$ = semiperimeter ABC

Using the law of sines, it can readily be shown that

$$(7) \qquad R_1=R_2=R_3=R.$$

1. The following relationship can be found by trigonometric means:

$$(8) \qquad r_1=4R\cos\frac{\alpha}{2}\sin\frac{\alpha+\beta-\gamma}{4}\sin\frac{\alpha-\beta+\gamma}{4}.$$

In this case—as in all subsequent ones—appropriate formulas for r_2 and r_3 can be obtained by cyclic permutation. Also:

$$r=4R\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$$

by a well-known formula.

2. An alternate relationship involving parts and only parts of the original triangle ABC is:

$$(9) \qquad r_1=\frac{c\sin\alpha\cos\beta\cot\gamma}{\sin\alpha+\cos\beta+\cos\gamma}$$

and, by a familiar formula:

$$r=\frac{2\sqrt{\dfrac{p}{2}\left(\dfrac{p}{2}-a\right)\left(\dfrac{p}{2}-b\right)\left(\dfrac{p}{2}-c\right)}}{p}.$$

B. The equilateral triangle deserves attention as a special case.

The following relationships suggest themselves:

$$(10) \qquad r_1=r_2=r_3=a-h$$

$$(11) \qquad \begin{aligned} r_1+r_2+r_3&=3(a-3r)=3(a-h)\\ &=3(2\sqrt{3}-3)r=(2\sqrt{3}-3)h\\ &=\frac{3(2-\sqrt{3})}{2}a. \end{aligned}$$

These formulas will obviously hold for AD, BE, CF being altitudes, medians, or angle-bisectors.

IV. Given *any* triangle ABC. Draw the three *medians*. Let the centroid be O. Relationship(s) between the radii of the circles inscribed in triangles COB, AOC, BOA, respectively, and the radius of the circle inscribed in triangle ABC are to be developed.

The same symbolism will be employed as in IIIA with the exception that F, D, E are now midpoints of AB, BC, CA, respectively, and point O thus designates the centroid.

Then $$r_1=\frac{2S_1}{p_1},$$

but $$S_1=\frac{S}{3} \quad \text{and} \quad S=\frac{rp}{2}.$$

Hence $$S_1=\frac{a+b+c}{6}\,r$$

$$\frac{p_1}{2}=\frac{3a+2(\overline{BE}+\overline{CF})}{6}.$$

If the law of cosines be applied to triangles CEB and ABC, respectively,

$$\overline{BE}=\frac{\sqrt{2(a^2+c^2)-b^2}}{2}.$$

Similarly $$\overline{CF}=\frac{\sqrt{2(a^2+b^2)-c^2}}{2}.$$

Thus

(12)
$$r_1=\frac{a+b+c}{3a+\sqrt{2(a^2+b^2)-c^2}+\sqrt{2(a^2+c^2)-b^2}}\,r.$$

V. Given *any* triangle ABC. Draw the three *angle-bisectors*. Let the incenter be O. Relationship(s) for the radii of the circles inscribed in triangles COB, AOC, BOA, respectively, and for the radius of the circle inscribed in triangle ABC can be found.

Using the same symbolism as before, F, D, E will now represent the points of intersection of the three angle-bisectors with the opposite sides, respectively, and point O will mean the incenter.

Applying the law of sines to $\triangle COB$:

$$\frac{a}{\sin\left(90+\frac{\alpha}{2}\right)}=2R_1.$$

Hence

(13a) $$R_1=\frac{a}{2\cos\frac{\alpha}{2}};$$

or also:

(13b) $$R_1=2R\sin\frac{\alpha}{2}.$$

Since

$$r=4R\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$$

is true for any triangle,

$$r_1=4R_1\sin\frac{\beta}{4}\sin\frac{\gamma}{4}\sin\left(45+\frac{\alpha}{4}\right)$$

and, with the help of formula (13a):

(14)
$$r_1=\frac{a\sqrt{2}\sin\frac{\beta}{4}\sin\frac{\gamma}{4}\left(\sin\frac{\alpha}{4}+\cos\frac{\alpha}{4}\right)}{\cos\frac{\alpha}{2}}.$$

The above investigations are examples of the great variety of relationships which can be found in this connection.

N.C.T.M. Summer Meetings

DETROIT, JUNE 30
Jointly with N.E.A.
See program on page 391 of this issue

EXETER, AUGUST 21-28
Jointly with New England Institute
See program in April issue, page 310

REFERENCES FOR MATHEMATICS TEACHERS

Edited by William L. Schaaf
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Logarithms and Exponentials

Cajori once pointed out that the miraculous powers of modern calculation are due to three inventions: the Hindu notation, decimal fractions, and logarithms. In the light of recent developments in the fields of electronic computing machines, cybernetics and mathematical communication, Cajori's dictum may need to be reconsidered. Nevertheless, the invention of logarithms is unique in several respects. It is one of those rare historical instances of an invention which was in no way adumbrated by the previous contributions of other workers. Moreover, the concept of a logarithm was arrived at without the aid of exponents, indeed, even before exponential notation had come into general use.

We can thus understand why the new creation was at first referred to as *mirifici logarithmorum canonis constructio.* An awareness of its significance is suggested by the following excerpts from the dedicatory letter written by Napier's son in the edition of 1619, two years after the death of his father:

"Several years ago (Reader, Lover of the Mathematics) my Father, of memory always to be revered, made public the use of the Wonderful Canon of Logarithms; but . . . he was decidedly against committing to types the theory and method of its creation, until he had ascertained the opinion and criticism on the Canon of those who are versed in this kind of learning. . . .

"You have then (kind Reader) in this little book most amply unfolded the theory of the construction of logarithms, (here called by him artificial numbers, for he had this treatise written out beside him several years before the word *logarithm* was invented,) in which their nature . . . are clearly demonstrated.

"We have also . . . printed some studies . . . on the new kind of logarithms by that most excellent Mathematician HENRY BRIGGS, public Professor at LONDON, who for the singular friendship which subsisted between him and my father of illustrious memory, took upon himself, in the most willing spirit, the very heavy labour of computing this new Canon, the method of its creation and the explanation of its use being left to the Inventor. Now, however, as he has been called away from this life, the burden of the whole buisness would appear to rest upon the shoulders of the most learned BRIGGS, on whom, too, would appear by some chance to have fallen the task of adorning this Sparta.

"Meanwhile (Reader) enjoy the fruits of these labours such as they are, and receive them in good part according to your culture. Farewell, ROBERT NAPIER, Son."

The teaching of logarithms is one of those areas in which perhaps unwittingly more emphasis is placed upon skills and manipulation than upon understanding. All too often the topic is learned, by itself, with little or no relation to other significant concepts outside that of exponents, and must be relearned a year of two later when the need arises. It is particularly unfortunate when students learn about logarithms without an appreciation

of their significance in science, technology, engineering, and astronomy, or without an understanding of the function $y=\log x$ and its relation to function $x=e^y$.

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Test Item for the "Gifted" Student!

Show that

$$\frac{\sin x}{n} = 6$$

APPLICATIONS

Edited by Sheldon S. Myers
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In the November 1951 issue of this department the opening discussion centered around the role of applications in mathematics instruction with some brief references to some of the literature on the subject. Our continued search and sifting of materials for this department has raised the fundamental and difficult questions: What are mathematical applications and how may they be classified? What should be their purposes in mathematics instruction? The answers to these questions are by no means obvious and present extremely difficult semantical and pedagogical problems. It should be clear to the open-minded that ultimate and final answers to the above questions cannot be given, since they involve value-judgments which depend upon one's philosophical position. Realizing this, we shall offer our own considered answers to the above questions. Furthermore we will welcome communication from you regarding alternative views.

It is our opinion that mathematical applications may be defined and classified by the following four-fold types:[1]

1. Discovery of a mathematical principle from specific problems or illustrations, such as the development by pupils of a rule for adding directed numbers from thermometer and football problems. (March issue, 1952)
   a. Examples should be sufficiently simple and concrete to be understood by pupils without the explicit formulation of the principle.
   b. Examples should be sufficiently varied to avoid pseudo-generalizations.
2. Borrowing a *method* from another context in order to develop a mathematical principle, such as illustrated by the application in this month's department.
3. Use of a mathematical principle in solving a problem in another context such as deriving the Fahrenheit-Centigrade relationship by means of parallel lines and proportion, solving numerical problems in mensuration by means of formulas, or proving geometric theorems by algebraic process.
4. Illustration of a mathematical principle or element in another context, such as the rigidity of triangles in construction, the speedometer as a differentiating and integrating device, the perfect trinomial square as an actual square with side $(a+b)$, snowflakes as hexagonal figures, the Fibonacci series in living forms.

The above types not only serve to define and classify but also suggest the pedagogical uses of applications. This brings us to the instructional purposes of applications. Here again the particular major and minor emphases which a teacher assumes with regard to applications depends upon whether his outlook is that of a humanist, an essentialist, a perennialist, a pragmatist, a reconstructionist, a realist, an idealist, a relativist, or an eclectic. Our opinion includes the following purposes for applications:

1. Utilitarian—involving needs, both adolescent and adult
2. Academic—promoting better mathe-

[1] Broadly speaking the "application" in (1) includes the whole discovery process, not the "example" alone; in (2) includes the entire borrowing procedure not merely the method; in (3) includes the purpose and method of use, not merely the problem.

matical understanding

3. Aesthetic—including appreciation, curiosity

We believe that motivation is an overarching purpose which may result from any one or all of the above purposes. Applications do not necessarily involve wit, lightness, and humor. When they do, they become difficult to distinguish from recreations. Dr. Aaron Bakst has an interesting and suggestive discussion around this point in the October 1951 MATHEMATICS TEACHER. Returning to the above purposes, it should be noted that these purposes can implement each other. There is a tendency to consider (1) above to be general education and (2) to be special education, while (3) can be both. A later paragraph in this issue discusses this point.

A narrowed outlook might result in persistent emphasis on only one or two of these purposes. On the other hand it is possible in a well-rounded program, for all of these purposes to be emphasized at one time or another. It is unfair therefore to criticize or condemn a particular application because it lays exclusive stress on one or another of these purposes, since it takes many applications of different types to make up a well-rounded program. It is also not sound to expect each application to be equally motivating to all pupils, since pupil goals and purposes vary greatly. Homogeneous grouping might permit a teacher to place greater stress on some of these purposes than others.

There are those who fear an increase in emphasis on applications in mathematics instruction on the grounds that the utilitarian purpose might take precedence over the mathematical and academic thus relegating mathematics to the role of handmaiden, rather than queen. This fear is augmented because of the view of some who hold that the only justification of mathematics in secondary education lies in its contributions to general education. Permit us to allay these fears with respect to this department. In the first place it would be impossible through job analysis or other survey techniques to uncover all of the utilitarian applications of mathematics. In view of this it is necessary to depend upon the cultivation of the *generalizing* powers of students which throws us back into the mathematical purpose as well as the utilitarian. We believe that special as well as general education are legitimate aspects of secondary education and that, in the words of the Harvard Report: " . . . the special flows out of the general and is forever returning to and enriching it."

Let us turn now to a discussion of the application referred to under "type two" application above, where a method of discovery is borrowed from another field and used in a mathematical context to develop a principle or rule of operation. The illustrated procedure must be employed with caution, since it is intuitive and non-rigorous. The following application is a continuation of the methods employed by OSCAR SCHAAF and described in the March issue of THE MATHEMATICS TEACHER.

Al. 16 Gr. 9 Developing a Rule for Multiplying and Dividing Directed Numbers by the Informal Use of a Method of Discovery in Astronomy

In the history of astronomy Halley's discovery of the comet now bearing his name is well known. Using the laws of Newton and the principles laid down by Newton for plotting the orbits of planets and comets, Halley plotted the paths of 24 bright comets appearing between 1337 and 1698 A.D. Halley selected three of these because their paths closely resembled each other and noted their dates of occurrence: 1531, 1607 (seen by Kepler), and 1682 (seen by Halley).

The interval of 75 or 76 years between these occurrences suggested to Halley that they might be the same comet. Acting on this hypothesis, Halley found in ancient records that a bright comet had occurred at each of the following dates in addition

to the ones listed above: 1305, 1380, 1456, 1531, 1607, and 1682.

Noting the continuity of form in the above dates and concluding these comet appearances were due to the same comet following a well-defined orbit, Halley concluded that this comet was due to return 75 or 76 years after 1682, which would be 1757 or 1758. However, taking into account the retarding influence which Jupiter would exert at that time, Halley predicted the comet would return towards the end of 1758 or the early part of 1759. The fulfillment of this prophecy after his death assured Halley a place among the immortals of science. Later appearances in 1835 and 1910 further confirmed Halley's prediction.[1]

After the students have considered this interesting method of discovery by generalization of a form or pattern, they could be asked to consider a similar technique in filling out the following sequence:

$$4\times3=12$$
$$4\times2=8$$
$$4\times1=4$$
$$4\times0=0$$
$$4\times-1=?$$
$$4\times-2=?$$
$$4\times-3=?$$

Practically all students are able to fill in this table on an informal basis. After the correct results have been obtained on this basis, they might be asked to generalize a rule for multiplying a positive and a negative number. Then the following sequence might be presented for consideration:

$$-4\times3=-12$$
$$-4\times2=-8$$
$$-4\times1=-4$$
$$-4\times0=0$$
$$-4\times-1=?$$
$$-4\times-2=?$$
$$-4\times-3=?$$

[1] Charles Patterson, *Problems in Logic* (New York: Macmillan & Co., 1926), pp. 301–4.

Again students are usually able to complete the table correctly by assuming a continuity of form. Similarly a rule is generalized for multiplying two negative numbers together.[2]

Of course once the rules for multiplying directed numbers have been developed, the rules for division of directed numbers follow as logical implicates of the former. To accomplish the development of the division rules by students, the following problems might be given:

1. $(+7)(?)=+35$
2. $(-8)(?)=-32$
3. $(-5)(?)=+30$
4. $(+4)(?)=-56$
5. $(-3)(?)=+93$
6. What process do you use to find the number inside the parentheses?
7. What do you think the rule for division of signed numbers should be?

Students might be asked to make up story problems to illustrate each kind of multiplication and division problem. Examples of these are as follows:

1. How much higher or lower was the temperature 6 hours ago if the temperature has (a) risen, or (b) dropped 5° each hour?
   a. $-6\times+5=-30°$ (lower)
   b. $-6\times-5=+30°$ (higher)
2. A football player lost 36 yards in 9 plays. If he lost the same yardage on each play, how many yards did he lose on each play?

$$\frac{-36}{+9}=-4 \text{ (yards lost on each)}$$

(*Partition division*)

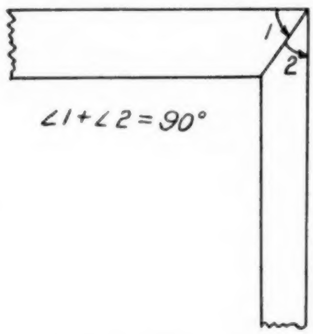
3. A football player lost 36 yards by a series of 4-yard losses. How many plays did it take?

$$\frac{-36}{-4}=9 \text{ plays} \quad (\textit{Quotition division})$$

[2] See Elizabeth M. Cooper, "Introducing the Multiplication Table for Signed Numbers." THE MATHEMATICS TEACHER. XLIII (December, 1950), 420–21.

P.G. 8 Gr. 8–11. *Complementary Angles in a Picture Frame*

In cases of picture frames whose horizontal pieces are wider than their vertical pieces, examples of unequal complementary angles occur at the joints as shown in the figure below.

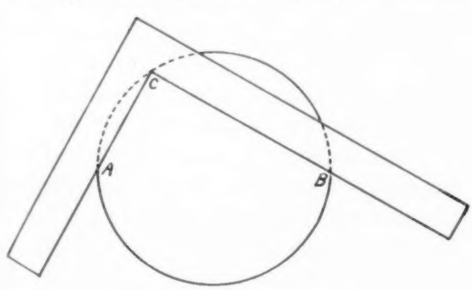


Complementary Angles in the Picture Frame.

For classes which have had the tangent function the following problem could be presented: compute angle 1 and angle 2 given the respective widths of the vertical and horizontal pieces of the frame.

P.G. 9 Gr. 8–11 *Drawing a Circle with a Carpenter Square*

A carpenter wishes to cut a round hole with diameter AB out of a panel with a coping saw. He decides to draw a circle on the panel, but finds no string and no compass handy. Undaunted, he made use of two nails and his carpenter's square and quickly drew his circle. How did he do it?

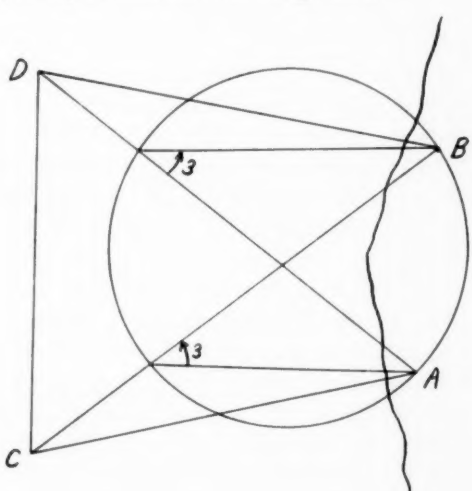


Drawing a Circle Without a Compass

He drove nails lightly into the panel at points A and B so that AB was the required diameter. Then he placed his square on the nails as shown above. By keeping the square snug against the nails and his pencil at vertex C, he rotated the square about the nails. Why does point C travel in a perfect semicircle? The lower half of the circle was obtained by flopping the square over and resting it against the under side of the nails.

P.G. 10 *Keeping a Safe Distance in Boat Navigation by Means of Inscribed Angles*

This is the last example from the Carter thesis. A boat captain is passing northward off-shore from two lighthouses A and B along a path from C to D. (See figure below.) It is necessary that he keep outside of the indicated circle because of hidden reefs. Taking his chart showing the lighthouses and the circle, he measured angle 3 and decided that if he steered a course so that the angle formed by his boat and the two lighthouses is less than angle 3, his boat would remain outside of the circle and thus be safe. Upon what geometric principles was his reasoning based?



Inscribed Angles Protect a Ship

NOTES ON THE HISTORY OF MATHEMATICS

Edited by Vera Sanford
State Teachers College, Oneonta, New York

A Problem from 16th Century Medicine

The following problem occurs in Jerome Cardan's *Practica arithmetice* which was published in 1539.

> A certain man mixes 1 ounce of medicine warm in the third degree, 3 ounces warm in the first degree, 4 ounces cold in the second degree, 5 ounces warm in the second degree, 2 ounces at temperate heat, 1 ounce cold in the fourth degree, 13 ounces cold in the first degree. He brews these together and makes one mixture from them. It is asked to determine the degree of heat or of cold of this medicine.

Other mixture problems were of frequent occurrence in the early printed arithmetics, but this one seems to have been unique with Cardan. The problem is interesting for a number of reasons. It appears to be a simple case of finding the temperature of a mixture which in itself is an anachronism for the first thermometer was not invented until some sixty years later. The problem is concerned with the mixture of medicines. Cardan had a variety of vocations and avocations. He had studied medicine at Pavia and he became professor of medicine at Pavia and later at Bologna. His reputation was so great that in 1552 he was invited to travel to Scotland to treat the archbishop of St. Andrews who was afflicted with asthma. It was no slight journey in the 16th century, but the archbishop is reported to have greatly benefited by Cardan's prescriptions. At any rate, he paid him handsomely. For fear that Cardan's skill be underestimated, it is worth noting that tradition has it that a part of the treatment consisted in the stipulation that his grace should thereafter use a hair pillow, from which we might assume that the patient was allergic to feathers. At any rate, Cardan was a physician of wide reputation, and presumably of good training for his day.

The key to the problem lies in the fact that the medicine involves the use of the word "gradus" (degree). According to Dr. Lynn Thorndike's *History of Magic and Experimental Science,* the theory of medicine, inherited from the Middle Ages and possibly from Galen himself, was that each drug had a certain number of degrees of heat or of cold. Each disease likewise had its degree of heat or of cold. Accordingly, a hot medicine should be prescribed for a cold disease and vice versa. By a proper mixture of drugs, a medicine of any desired degree of heat or of cold might be obtained. It would appear then that Cardan was drawing on his medical knowledge to frame this problem in mixtures which might quite conceivably have been standard practice when a physician was obliged to concoct a medicine of a stated degree of temperature using the drugs he had at hand.

In solving this problem, Cardan multiplies each number of ounces by its assigned degree of heat or of cold. He finds the sum of the hot products, and the sum of the cold ones. There is a proviso that the degrees are equally spaced. He then combines these two sums, subtracting the smaller number from the larger and using the name (hot or cold) of the larger number. This result is then divided by the total number of ounces, being sure to include those of neutral temperature. The quotient is the temperature of the mixture. It will be noticed that Cardan is using a

(*Continued on page 372*)

DEVICES FOR A MATHEMATICS LABORATORY

Edited by Emil J. Berger
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This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are requested to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning Devices for a Mathematics Laboratory to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

A Chart of Equivalents

This device is useful as a teaching aid in helping pupils develop an understanding of common fractions, decimal fractions, per cents, and the ways in which these concepts are related. A properly constructed device can be manipulated by both teachers and pupils with ease. By keeping the chart in an easily accessible place in the classroom it will be available for handy reference whenever needed.

The device consists of three main parts: a piece of plywood, a rail, and a runner. The dimensions of the sheet of plywood are $\frac{3}{8}'' \times 18'' \times 46''$. Prepare the board for ruling and labeling by giving it two coats of shellac. This will prevent the ink from running with the grain of the wood. Rule and label the board as illustrated in the diagram. (See Fig. 1.) The ruled scale at the top of the board is one meter long and is divided into centimeters. Every tenth division is indicated with a heavy black line. In preparing the device it might be better to draw these "heavy" lines with red ink. Doing this will make it much easier to locate the "tenths."

To construct the rail which is to be fastened to the board above the centimeter scale procure a strip of fir $\frac{1}{2}'' \times 1'' \times 46''$. Plane one long edge down so that the shape of its cross section will look like

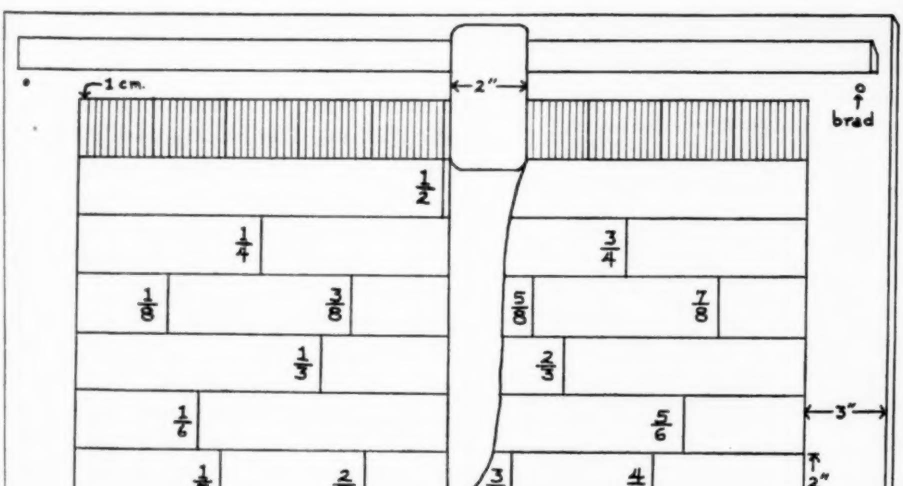

Fig. 1.

Fig. 2.

a trapezium with parallel sides of 1″ and $\frac{5}{8}$″. (See the notch in the runner illustrated in Fig. 2.) Fasten the rail to the plywood so that the narrow face will be next to the plywood. For the most efficient operation of the runner the rail should be fastened with wood screws inserted through the back side of the plywood sheet.

The construction of the runner is illustrated in Figure 2. The reader will note that the notch cut in the top of the runner must fit the rail. Also the part in which the notch is cut must be wide enough so that the rail and the runner will always be at right angles to each other. To keep the runner from sliding off the ends of the rail small brads should be driven into the plywood near the edges below the rail as shown in Figure 1. A coat of varnish and a coat of paste wax applied to the surfaces of the board, rail, and runner will insure easy movement of the runner when it is moved along the rail.

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AN AREA FINDING DEVICE

In industries like newspaper and photo-engraving where areas of rectangular figures must be found rather frequently methods have been devised for finding such areas without actually performing the mathematical computations involved. A description of the construction and use of the apparatus illustrated in Figure 3 will show how this can be accomplished.[1]

The device consists essentially of the graph of the first quadrant branches of the family of rectangular hyperbolas whose equations are $xy=k$, where k is a positive constant. The asymptotes of this family of curves are, of course, the coordinate axes.

[1] In a letter to the department editor, George R. Anderson of Millersville State Teachers College reports that he saw the device described in this article being used in a print shop in Williamsport, Pennsylvania.

In order to draw this family of hyperbolas any mechanical device for drawing hyperbolas may be used varying the constant k (in a systematic arrangement) for each case. However, the curves can also be drawn free-hand by plotting points for each curve. This is the way in which the graph shown in the illustration was completed.

To construct the device draw a square 20″ on a side on a large sheet of construction paper and divide it into 1″ squares. Plot each hyperbola separately, draw it in with pencil, and ink it with India ink. The values used for k in the model illustrated are 1, 2, 4, 6, 8, 10, 15, 20, 25, 30, and all multiples of 10 from 40 through 400. It will be noted that the value of k for each curve appears at the right-hand end of the full drawn curve. When the graph is completed mount it on a piece of fiber board and tack on two wooden venetian blind strips so that their inner edges coincide with the x and y axes. Thus the wooden strips can be made to serve as a guiding frame for aligning rectangles whose areas are to be found.

To use the device for finding the area of a rectangular object such as a sheet of paper, book, block of wood, etc., place the object on the device so that a corner of the object coincides with the origin of the graph and the two sides adjacent to this corner lie along the edges of the guiding frame. By looking at the corner diagonally opposite the one at the origin and locating the curve it touches or is nearest to, the observer can find the area of the object approximately by reading the number at the right-hand end of the curve.

As an example, suppose that a sheet of typing paper ($8\frac{1}{2}$″×11″) is placed on the device so that one corner of it is at the origin and the edges of the paper adjacent to this corner lie firmly against the guiding frame. By looking at the corner of the sheet diagonally opposite the origin, the observer can see that it falls between the curves marked 90 and 100. A little careful observation will reveal that if a

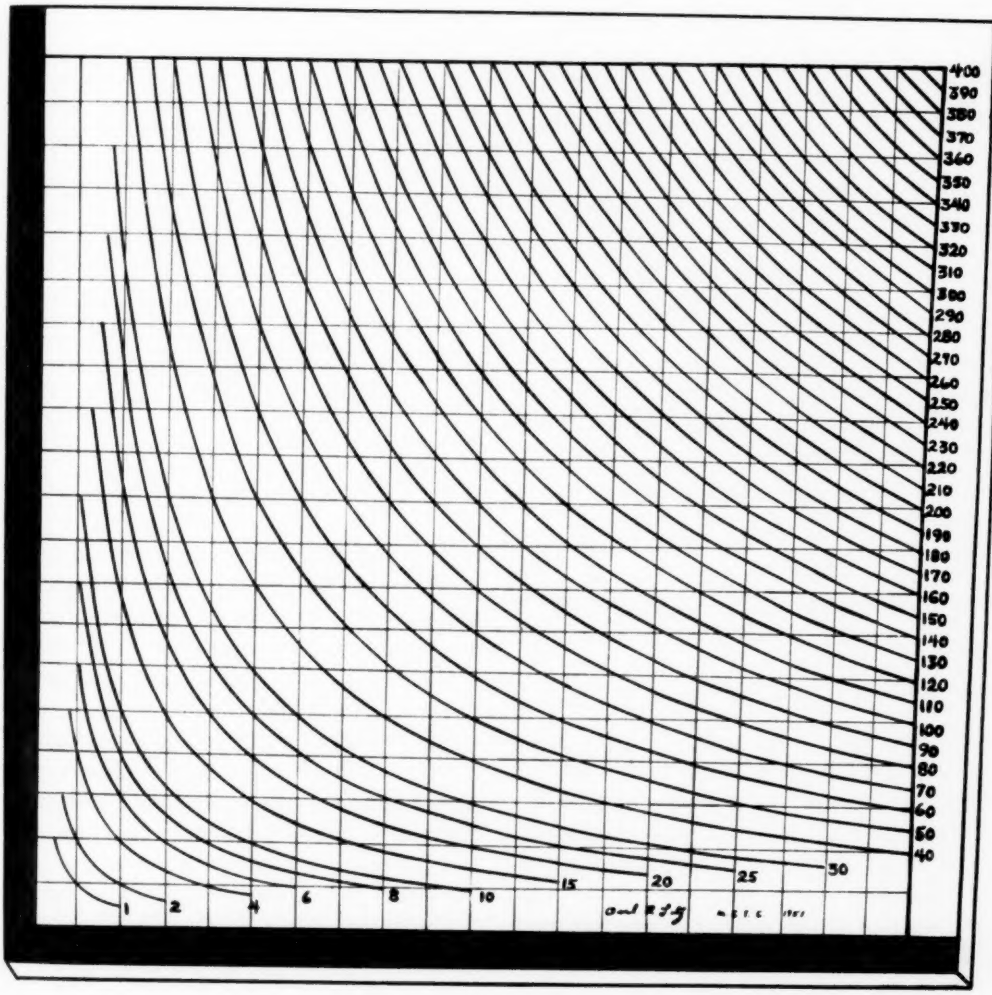

Fig. 3

curve had been drawn with k equal to 93 or 94 the corner of the sheet would probably have been on it. Hence it can be estimated that the area of the sheet is somewhere between 93 and 94 square inches. ($93\frac{1}{2}$ square inches is the correct result.) Thus a fairly accurate estimate can be obtained quickly and easily.

Carl R. Foltz
Millersville State Teachers College
Millersville, Pennsylvania

Pythagorean Theorem Model

Illustrated in this article is a set of models which can be used to verify or demonstrate informally the truth of the Pythagorean Theorem from the point of view of area. By using the models it can be shown that the squares on the two legs may be made to fill exactly the area of the square on the hypotenuse.

The set of models consists of a right triangle with squares mounted on each side, duplicates of the squares on the two legs, and another duplicate of the smallest square divided into rectangles and squares as indicated below. All figures may be made of heavy cardboard or plywood.

To be specific, suppose that the sides of the right triangle are respectively 10, 8, and 6 units. Then,

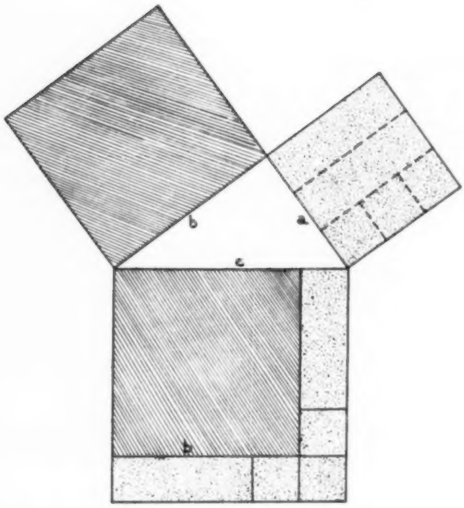

FIG. 4.

$6^2=10^2-8^2=(10-8)(10+8)=(2)(18)$;

and

$6^2=(2)(6)(3)$.

The three factors in the right member of the last equality indicate the manner in which the square on the smaller leg is to be divided. According to the scheme it is to be divided into 3 rectangles each 2 units wide and 6 units long. Note that the lengths of the rectangles are the same as the side of the square. Finally divide one of the rectangles into squares. There will be three, each 2 units on a side. The two leg squares will now fill exactly the area of the square on the hypotenuse. (See Fig. 4.)

A similar procedure may be used for a right triangle with sides of 17, 15, and 8 units:

$8^2=17^2-15^2=(17-15)(17+15)=(2)(32)$;

and

$8^2=(2)(8)(4)$.

In general, for any right triangle ABC with c as the hypotenuse and $b>a$,

$$a^2=c^2-b^2=(c-b)(c+b);$$

and

$$a^2=(c-b)(a)\left(\frac{c+b}{a}\right).$$

The three factors on the right indicate the manner in which a^2 (the square on the smaller leg) is to be divided; $(c-b)$ is the width of the rectangles; (a) which is the same as the side of the square will be the length of the rectangles; and $(c+b)/a$ is the number of such rectangles.

Any rational Pythagorean numbers may be used for easy construction of this model. Obviously, it is much easier to lay off the larger leg square on the hypotenuse and then divide the smaller leg square to fill the remaining area. However, it may also be done the other way.

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History of Mathematics

(*Continued from page 368*)

scheme that closely resembles our method of finding the arithmetic mean of several numbers when the computer uses an assumed mean. Cardan does not mention plus or minus numbers. He calls them numbers that are hot and numbers that are cold.

Cardan's idea of degrees of heat and cold and a point of neutral temperature had a strong resemblance to our idea of a thermometer with a zero at an arbitrarily designated point and temperature read above and below this point. This interpretation is not justified. All that can be said with certainty is that Cardan drew on medicine as he knew it to provide an illustration of a way that a physician *might* compound his drugs. We cannot even assume from this instance that any one even Cardan himself actually did this. He used a situation that might happen, and in doing so came very close to a method we apply to other cases.

AIDS TO TEACHING

Edited by

Henry W. Syer
School of Education,
Boston University,
Boston, Massachusetts

and

Donovan A. Johnson
College of Education,
University of Minnesota,
Minneapolis, Minnesota

BOOKLETS

B. 101—History of Mathematics in Cartoons

Mel Lieberstein, 434 North Pearl Street, Lebanon, Illinois.

Booklet; $8\frac{1}{2}'' \times 11''$; 40 pages; \$1.50.

Description: The upper part of each page in this book is a cartoon and the lower part a short synopsis of some part of the history of mathematics. The mimeographed pages are bound together in cardboard covers in a spiral binding. The whole book is in black and white only.

Appraisal: The idea of enlivening mathematics instruction with cartoons and enriching it with history is an admirable one. The language and degree of generality are certain to appeal to high school pupils. Unfortunately, the book is not very attractive, there are liberties and even inaccuracies among the mathematical and historical facts, and sometimes a flippancy when only a light-hearted approach is necessary. As an inspiration to make other cartoons and to delve into history these cartoons are excellent; as a source of information they are misleading.

B. 102—Trigonometry Tables

Illinois Tool Works, 2501 North Keeler Ave., Chicago 39, Illinois.

Booklet; $3'' \times 6\frac{1}{2}''$; 52 pages; One copy free to mechanics, shop men, engineers and teachers; \$1.50 for 20 copies.

Description: There are four pages of trigonometric formulas, one page of decimal equivalents, one title page, one page of advertising and the rest is devoted to five place tables of all six functions, tabulated to each minute of angular measure.

Appraisal: To contain five place tables in such a small booklet means that each entry must be in very small print which would be very tiring to use for any length of time. However, this is the least expensive set of five-place tables now available and is excellent for reference and supplementary use in any class.

B. 103—Mathematical Pie

Mr. R. H. Collins, Gateway School, Leicester, England.

Periodical; $5\frac{1}{2}'' \times 8\frac{1}{2}''$; usually 8 pages; 2 d. per copy, plus postage for less than six copies.

Description: This is an attractive and useful publication which is aimed directly at the pupils in secondary mathematics. It is filled with illustrations and contains very short articles on mathematical ideas, cross-word puzzles, a mathematical atlas on the history of mathematics, games, problems, and mathematical fallacies.

Appraisal: Even though much material of interest to pupils of secondary mathematics can be found in various places, there is a great deal of value in having such items collected in a periodical. This type of publication has long been needed; it is good to see such a splendid job turned out. The editor says that arrangements are being sought to distribute this in the United States also and we certainly hope this can be done. It should have a waiting audience.

Until then, send your requests to England and include an international money order in advance to cover a handful of copies; you will not regret it.

FILMSTRIPS

FS. 108—Fractions, Decimals, and Percentage

The Jam Handy Organization, 2821 East Grand Boulevard, Detroit 11, Mich.

B&W ($4); 59 frames.

Description: Frames 3–23 deal with definitions of fractions and decimals. They answer the questions: How big? How much? How many? When we find the number of units and a "little bit more," fractions and decimals are used to express the result. The meaning of fractions having various denominators is shown. When the total number of parts is 10, 100, or 1000 we use a simpler form called a decimal. Frames 24–28 show us how to convert fractions to decimals. It is important to do this when fractions and decimals are in the same problem. Arithmetic is easier when we use decimals. Division of the numerator by the denominator is shown. Frames 29–36 explain mixed and improper fractions. In problems, mixed numbers should be changed to improper fractions to avoid confusion. When results are improper fractions, they should be changed to mixed numbers. Frames 37–59 present percentage which is another system to show a "little bit more." Percentage is a fraction with 100 parts in the denominator. Percentage shows the comparative size of things. It is difficult to visualize 14/70 of 125 airplanes, whereas it is easier to visualize 20% of 125 airplanes. We also see an illustration of 150%. In all problems in percentage the same three elements are present—the %, the part, and the whole. We must always know two of the three to find the third. The last few frames are devoted to the use of the per cent "wheel." This is a diagram which, by revolving, shows how to find the part, the whole, or the per cent.

Appraisal: This filmstrip was shown to classes in general mathematics. It is an excellent refresher on the meaning of fractions, decimals, and percentage. It explained what each is but did not show any of the fundamental processes involving them. The reviewer liked very much the treatment of percentage. Although the per cent wheel clearly shows the relationship of the four parts of any per cent problem—the part, the whole, the per cent, and 100—its usage tends to make the solution of problems too mechanical. This is a fine filmstrip for classes in General Mathematics. It also can be used as a refresher in any class in high school mathematics. (Reviewed by Herbert Freed, Senior High School, Atlantic City, N.J.)

FS. 109—Order of Operations

The Jam Handy Organization, 2821 East Grand Boulevard, Detroit 11, Mich.

B&W ($4); 46 frames.

Description: Frames 3–19 explain how to plan a problem. This involves doing everything required, including proper units, and saving unnecessary operations. Always determine what is given and what is to be found. Frames 20–25 treat problems involving only opposite operations such as $25\times18\div5\times7\div21$ and $5+3-15+7-6$. Opposite signs are explained. Frames 26–38 deal with problems involving all operations. We are told that signs not the same or opposite are danger signals. Parentheses or bars under the terms are used to group plus and minus situations and multiplication and division situations. The statement is made that it is agreed that we do multiplication and division first. Frames 39–46 are concerned with short cuts. In conclusion it is stated that it pays to plan the problem before working it.

Appraisal: This filmstrip was shown to classes in general mathematics. It brought forth many class questions and discussions. The reviewer liked the manner in which a

practical problem is carried through the various operations. Although algebraic expressions were not used in the explanation, a student should understand "Order of Operations" if he observes this filmstrip attentively. One objectionable explanation in the filmstrip was the use of the word cancel, for division. (Reviewed by Herbert Freed, Senior High School, Atlantic City, New Jersey.)

FS. 110–FS. 117—Fraction Series

FS. 110 The Meaning of Fractions (45 frames)

FS. 111 Changing the Terms of Fractions (46 frames)

FS. 112 Adding Like Fractions and Mixed Numbers (47 frames)

FS. 113 Subtracting Like Fractions and Mixed Numbers (37 frames)

FS. 114 Adding Unlike Fractions and Mixed Numbers (53 frames)

FS. 115 Subtracting Unlike Fractions and Mixed Numbers (44 frames)

FS. 116 Multiplying Fractions and Mixed Numbers (62 frames)

FS. 117 Dividing Fractions and Mixed Numbers (47 frames)

Society for Visual Education, 1345 Diversey Pkwy., Chicago 14, Ill. or Stanley Bowmar Company, 513 West 166th Street, New York, N. Y.

B & W ($3.25 each, $24.00 for the set).

Description of FS. 111: This strip shows how to change fractions to higher terms, how to change fractions to lower terms, and why we reduce fractions. A circle is divided into sections to show equivalent fractions such as $1/2=3/6$ and $3/4=12/16$. Instructions on how to change fractions, by multiplying or dividing numerator and denominator by the same number are related to the diagrams. The last eight frames consist of review and practice questions.

Description of FS. 112: This strip shows the meaning of like fractions, improper fractions, and mixed numbers. The addition of like fractions and mixed numbers is illustrated concretely and worked out numerically. This addition frequently requires changing the answer from an improper fraction to a mixed number; several questions and practice problems are supplied in the strip.

Description of FS. 113: This strip illustrates with everyday situations and objects and numerical examples how to subtract like fractions, mixed numbers and whole numbers from mixed numbers. The examples do not require denominators to be changed to common denominators before subtracting. Practice is provided for the viewers by giving examples and asking questions.

Description of FS. 114: After illustrating the addition of peaches to show the addition of like fractions and the addition of a gallon and two quarts to show the addition of unlike fractions, this strip consists almost entirely of examples worked on a blackboard. Questions and examples for the viewer to complete are included.

Description of FS. 115: Of the 43 frames of this strip 19 are blackboard examples, 7 are title frames, and 17 are illustrations of the meaning of fractions in terms of drawings or objects. The examples on the blackboard and the illustrations show how to subtract unlike fractions, how to subtract mixed numbers, how to subtract fractions from whole numbers, and where the subtraction of unlike fractions and mixed numbers are used. Practice examples and questions are included.

Description of FS. 116: This strip includes seven title frames, 32 frames of blackboard work, and 23 illustrations of the meaning of multiplying fractions and mixed numbers. The illustrations include pictures of apples, circles, distances, cake, pie, boards and a cornfield. The blackboard examples show how to multiply fractions by whole numbers, fractions by fractions, mixed numbers by fractions, and

mixed numbers by mixed numbers.

Description of FS. 117: This strip includes 8 title frames, 12 frames of blackboard examples, and 27 frames illustrating the meaning of division of fractions and mixed numbers. These illustrations include such common objects as cake, pie, cheese, a farm field, and the distance between two cities. Division is shown to be accomplished by inverting the divisor and multiplying. This process is introduced by showing that the division of a whole number by a fraction with unit numerator always gives a result that is equivalent to multiplying the whole number by the denominator of the fraction.

Appraisal of FS. 110–FS.117: This series of filmstrips will furnish the teacher with illustrations of how to make fractions meaningful. The series does well in relating the concrete illustration to the method of working examples with symbols. It is unfortunate that such a large proportion of the frames of some of the strips must be devoted to the solving of examples. To date no one has devised a way to avoid the need for this abstract material. Thus, most of the material found in these strips is also found in modern arithmetic textbooks. Whether or not the dramatic presentation by projection results in more effective learning needs to be tested by research before judgment is rendered.

FS. 118–FS. 123—Mathematics Series

FS. 118—Thinking in Symbols (27 frames)

FS. 119—Grouping Symbols and Order of Operations (34 frames)

FS. 120—Geometric Figures (29 frames)

FS. 121—Measurement (36 frames)

FS. 122—Variables and Coordinates (33 frames)

FS. 123—Mathematics in Daily Living (28 frames)

McGraw-Hill Book Company, 446 West 42nd Street, New York City

B & W ($5.00 each or $27.00 for set of 6).

Description of FS. 118: This filmstrip introduces the symbols of algebra by relating them to familiar symbols such as railroad and highway signs, flag signals, and numbers. Symbols are shown to be a rapid and accurate way to present ideas. The symbols of algebra such as letters, signs of operation, abbreviation, parenthesis, and formulas are used to illustrate the convenience of using them.

Description of FS. 119: After showing the need for a specific order in operating on arithmetic problems containing addition and multiplication, the procedure of performing multiplication and division, before additions and subtractions is applied to algebraic expressions. This leads to a discussion of the need and convenience of grouping symbols to avoid confusion. The combination of abstract numbers and concrete numbers is used to show the combination of like terms in algebraic expressions. The filmstrip ends by summarizing the commutative and distributive laws of multiplication and the power of algebra to generalize.

Description of FS. 120: This filmstrip uses common articles such as a carpenter's square and an automobile cylinder to illustrate geometric facts. The carpenter's square is used to show how to draw a perpendicular line and bisect an angle. The wheel of the automobile is an example of a circle with a certain center, radius, diameter, and circumference. The cylinder of the automobile engine has a certain bore, stroke, and displacement which furnish a base for computing the volume of a cylinder.

Description of FS. 121: The measurement of the distance between two towns by two motorists and a highway engineer illustrate how measurement is always approximate. The most accurate measurement is shown to be the average of several "correct" measures. The errors occurring in the use of a ruler illustrate the need for a micrometer for more accurate linear measures. The operation and reading of a micrometer are briefly explained. The use

of a scale by a jeweler, a grocer, and a coal dealer is used to show the need for various units of measure, various degrees of accuracy and the significance of a given weight. The rounding off of numbers and the rule to follow in the multiplication of approximate numbers are illustrated. The filmstrip emphasizes that every measured quantity is an approximation whose accuracy is dependent upon the measuring instruments used.

Description of FS. 122: A trip between two towns furnishes data to illustrate relationship and dependence. These data are then applied to the formula $d=rt$ to show the meaning of variables and constants. A graph of the data illustrates how to draw a graph, the terms used in graphs, and the nature of a graph of an inverse variation. The graph of an airplane climbing at a constant rate of 15 degrees is used to show direct variation.

Description of FS. 123: This filmstrip gives examples of the use of mathematics by a housewife, by a community, and by an industry. The problem of buying a rug to fit a room as well as a budget is the basis for the application of mathematics in the home. The solving of this buying problem consists in rounding off measurements, converting feet to yards, computing area, and cost, and drawing to scale. The buying of a fire engine involves computing interest, sharing the cost, and showing the purpose of taxation. The problem of a car-dealer meeting a sales quota is solved by graphing.

Appraisal of FS. 118–FS. 123: The filmstrips, Geometric Figures and Mathematics in Daily Life, and Measurement, will furnish the mathematics classroom with realistic problems of significance and interest to ninth graders. The discussion of the automobile cylinder which includes information about the explosion in the cylinder as well as relating the dimensions to terms such as stroke, is the kind of presentation of the whole situation which is in harmony with principles of learning. The other three filmstrips contain much abstract material using symbols and presentation similar to textbook or blackboard treatment. The filmstrips are usually short, about thirty frames, and, thus, cover much material in a brief time. Thus, they will probably be most useful for review or summary.

INSTRUMENTS

I. 35—Blackboard Compass

The Mathaids Company, 336 Kirk Avenue, Syracuse 5, N.Y.

Compass; $1\frac{1}{4}'' \times 15''$; \$2.25.

Description: A wooden stick is marked along an edge so that circles with radii from $1''$ to $12\frac{1}{2}''$ can be drawn by adjusting a movable chalk holder. The other end contains a fixed pivot.

Appraisal: Why must people always invent complicated devices for doing simple jobs. This device is very ingenious, but a piece of string, a ruler, and a bit of practice is much better equipment. Specifically this is complicated by a tricky device for holding a piece of chalk which must be of a certain size to work, a rubber center to the pivot which breaks off easily and makes the device worthless, and an excessive amount of machining for a simple instrument. Ingenuity should be spent in making for teachers devices which they cannot make for themselves.

Note 1. When the Chinese Abacus (I.30, March 1951 issue) was reviewed, only the smaller 5-column model had been seen. Since that time Mr. Loy has sent us one of the larger 11-column models. It shows the same excellent workmanship as the smaller model. By all means write to Mr. W. D. Loy, 1317 Rhode Island Avenue, N.E., Washington 18, D. C. for information about these models. They are authentic, useful and decorative; they are more than toys; they are sturdy practical computing devices. At an elementary level they can introduce number concepts, at higher levels they can introduce new methods of calculation.

Note 2. A correction should be noted for the Hand Sighting Level recently reviewed (I.33, November, 1951 issue). The name of the manufacturer should be Swift and Anderson, but the address is correct. We quoted the price as \$2.00 and a letter from the company has asked us to change this to \$2.50 with an institutional discount of 20% to schools.

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

53. A Device for Teaching Logarithms

All of us, in the introduction and teaching of logarithms, have stressed the fact that the logarithm is merely an exponent. Many texts have short tables of the form of $2^x=n$, or $3^x=n$ to show the use of logarithms. Later on the existence of an exponent as a whole number plus a fraction is demonstrated more or less by the inductive method. Thus if $100=10^2$ and $1000=10^3$, then $350=10^{2+\text{a fraction}}$. This seems very clear at the moment and for a short time thereafter, but all of us in later work have had to answer numerous questions as to where to place the decimal point in the antilogarithm or what the characteristic should be in getting the logarithm of a given number.

I would like to describe a certain device that I have used for some time which has reduced the number of questions and has made the study of logarithms a meaningful activity on the part of the pupil. It has also served as a very useful means of understanding the principle and use of the slide rule.

Multiplication and division are rapidly reviewed to justify heuristically the definition $a^0=1$ by the usual method of showing that $a^3/a^3=a^{3-3}=a^0$ and $a^3/a^3=1$, and concluded by emphasizing that $10^0=1$. This is followed by $\sqrt{a^6}=a^?$, $a=a^?\cdot a^{1/8}$ and $\sqrt{a^{1/2}}=a^{1/4}$. After this has been fully understood the class is now ready to make Figure 1, which is the start of the final table shown in Figure 6.

This is followed by placing $10^{1/2}$ on the table (Figure 2) and asking the class what the relationship is between $10^{1/2}$ and 10^1. A large per cent of the class will recall that $10^{1/2}$ is the square root of 10^1 from the previous discussion. To obtain the number corresponding to $10^{.500}$, a few of the class will answer correctly that the square root of 10 must be taken. This presents an opportunity to review the extraction of square root, or to teach it to those who have never learned it. When 3.16 is obtained it is placed on the chart as shown in Figure 3.

The question is then put to the class as to how to obtain other points on the scale. Since $\sqrt{a^{1/2}}=a^{1/4}$ is still on the board,

FIG. 1

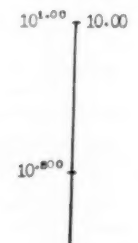

FIG. 2

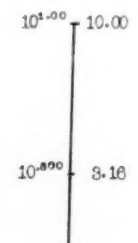

FIG. 3

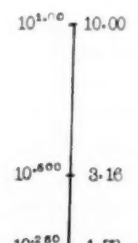

FIG. 4

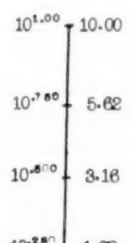

FIG. 5

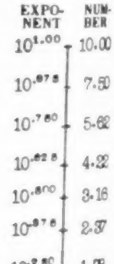

FIG. 6

or because some recall the development, the point $10^{1/4}$ or $10^{.250}$ is placed on the scale and the corresponding square root of 3.16 is computed (Figure 4).

A leading question as to what two powers multiplied will result in $10^{3/4}$ will get the response that $10^{1/2}\times 10^{1/4}=10^{3/4}$. Someone will suggest that the corresponding antilogarithm can be obtained by getting the product of 3.16 and 1.78 (Figure 5). The same procedure will obtain $10^{1/8}$, $10^{3/8}$ $10^{5/8}$, $10^{7/8}$ and their corresponding antilogarithms (Figure 6). A review and closer examination of these derived tables will give the pupil a better understanding of the mantissa than will be given by telling him that if $10^0=1$ and $10^1=10$ then 8 is 10^x where x is between 0 and 1.

The class is now ready for multiplication. The product of 3.16 and 1.78 can be obtained orally by the class by use of Figure 6, after the pupils are shown that all that is necessary is to add the exponents and to find their sum and its corresponding antilogarithm further up on the scale. Numbers are selected so that the exponent of the product does not exceed 1. When the pupils are thoroughly familiar with the use of the table, a problem is selected which will "run off" the scale. The need for another table is then shown.

The new table (Figure 7) is made with the limits of 10^1 and 10^2. $10^{1\ 1/2}$ or $10^{1.5}$ is placed on the scale. By asking the class which two powers multiplied together will give $10^{1\ 1/2}$ or $10^{1.500}$, several combinations will be suggested, such as $10^1\times 10^{.500}$, $10^{.750}\times 10^{.750}$, etc. The product of the corresponding antilogarithms will lead to

Fig. 7

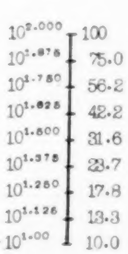

Fig. 8

Fig. 9

the same resulting product, 31.6. The class will spend some time completing the table shown in Figure 8. Several members of the class will notice that Figures 6 and 8 have the same corresponding significant figures. Some will notice and all will be somewhat surprised to see that the only difference is the position of the decimal point. After the table is made with limits of 10^2 and 10^3 (Figure 9), the relationship between the whole number part of the exponent (the word "characteristic" should not be used at this time) and the normal position of the decimal point in Figure 6 is clearly demonstrated.

A great number of multiplication problems are placed on the board using only those numbers found on the tables, (Figures 6, 8, 9) such as $1.78\times 5.62\times 3.16$ and arranged as follows:

$$\begin{aligned}17.8&=10^{1.250}\\5.62&=10^{.750}\\3.16&=10^{.500}\\\hline &\quad 10^{2.500}\end{aligned}$$

The pupils will then be asked to do similar examples using the first table (Figure 6) only. The class will soon see that one table only is sufficient no matter how large the numbers are. When pupils ask how to find the products of numbers not found on our tables, I tell them that later on we shall use a table which has all the numbers consisting of three significant figures.

Until now, negative exponents have been avoided in this development. They are now quickly reviewed. Pupils are then asked to write $.178=1/10$ of $1.78=10^{-1}\times 10^{.125}$; $.00316=1/1000$ of $3.16=10^{-3}\times 10^{.500}$; and many others. The above seldom presents any real difficulty even when pupils are asked to write $10^{-3}\times 10^{.500}$ as $10^{\bar{3}.500}$. The table shown in Figure 10 is developed slowly with the help of the class. The class is then asked to complete Figures 11 and 12.

The pupils can perform multiplication such as $.0136\times 7.50$ by arranging their work in column form as previously done.

After several problems similar to the

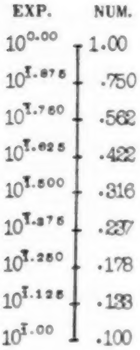

FIG. 10

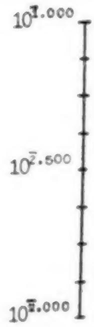

FIG. 11

FIG. 12

above have been done by the class, it will become apparent that the tables shown in Figures 10, 11, and 12 are not necessary, and that the table shown in Figure 6 may be used for all the multiplication problems providing they watch the characteristic. I refer to the table shown in Figure 6 very frequently as the *normal position table*.

Division of powers is quickly reviewed and pupils are asked to perform division by the use of the tables. This presents no difficulty if the problems are selected so that all the characteristics are positive. Then problems involving multiplication and division are assigned; such as

$$\frac{3.16\times 1.78}{4.22}.$$

The class is now introduced to the logarithm tables printed in the textbook. When asked to find the logarithm of 3.16 they are pleased to find the value .4998 in almost perfect agreement with the values in our derived table. They derive similar satisfaction in checking the other values of our table.

The presentation of logarithms as described in this article does not take more classroom time than the usual development found in most textbooks. It is my feeling that this device is concrete and understood in theory as well as in practice by the pupils. Several pupils, in making slide rules, glued strips of paper having the tables shown in Figure 6 on both the slide and the stationary part of the rule. Since all the pupils constructed the tables shown in Figures 6, 8, and 10, they have an idea of what is involved when they are using the tables of logarithms.

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Editor's Note: Rather than the table of values based on $2^x=n$ or $3^x=n$ mentioned by Mr. Prag, I like to begin with $4^x=n$, because then students can readily fill in the table for $x=1/2, 3/2, -1/2$, etc. This might still be done before introducing $10^x=n$ as does Mr. Prag.

In connection with Mr. Prag's remark about student made slide rules, I should like to suggest that the construction of slide rules may well be preceded by the use of a pair of dividers or compasses with a single log scale ("Gunter's Scale") for its historical interest as the forerunner of the slide rule, for its pedagogical value as a simple step introductory to the slide rule, and for its interesting current use on various types of Navy navigation charts. (see "Miscellanea—Mathematical, Historical, Pedagogical," THE MATHEMATICS TEACHER, vol. 42 (Oct. 1949) p. 307.

The students should first make their own log scales and perhaps even simple slide rules, but then more accurate rules may be made rapidly in class by cutting several sheets of semi-logarithmic graph paper into narrow strips with a paper cutter. Two strips placed on a students desk and labeled by the student, may be slid back and forth on the desk to operate the rule. The student may then at home use the scale from the strips or mount the strips themselves on wood or cardboard to make slide rules.

This use of a special coordinate paper may also arouse interest in special papers and in other uses of semi-logarithmic and logarithmic paper. Here can be born several projects.—P. S. J.

54. More About Cevians, Nedians, Redians

Following the publication of *Miscellanea 32, 34* and *37*, several letters on this topic were received. We publish here excerpts from two of them. The first of them uses a different theorem as a lemma for its approach to the old formulas. The second uses only an elementary theorem known to all high school students.

Mr. Alan Wayne, in the November, 1951 number of THE MATHEMATICS TEACHER comments on *Cevians, Nedians,* and *Redians.* He shows how some well-known theorems of Geometry (Ceva's and Menelaus' and their respective converses) arise from two expressions which he

gives relating to the Cevic and the Menelaic Triangles. Mr. Wayne remarks, "By analytic geometry—making use of the ratio formula, and the determinant form of the area of a triangle—it is readily shown that":

$$\frac{\triangle PQR}{\triangle ABC}=\frac{(1-rst)^2}{(1+r+rs)(1+s+st)(1+t+tr)}$$

$$\frac{\triangle DEF}{\triangle ABC}=\frac{1+rst}{(1+r)(1+s)(1+t)}$$

The above expressions may be arrived at using only elementary methods of algebra and geometry. We submit here a derivation of the first expression only as it is rather lengthy.

We base our derivation on a simple property of the triangle which is given in Durell's *Modern Geometry* as a theorem. The theorem, and the proof as given by Durell is as follows: (Figure 13)

"Two triangles ABC, ABD have a common base AB; The line joining their vertices cuts AB at X; Then

$$\frac{ACB}{ADB}=\frac{CX}{DX}.$$

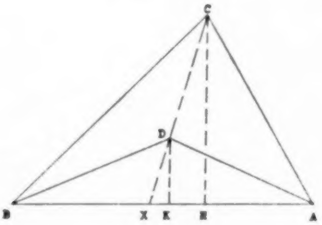

FIG. 13

Proof: Drop perpendiculars CH, DK to AB. Then:

$$\frac{\triangle ACB}{\triangle ADB}=\frac{(\frac{1}{2})AB\cdot CH}{(\frac{1}{2})AB\cdot DK}=\frac{CH}{DK}=\frac{CX}{DX}$$

by similar triangles.

Applying the above result to Figure 14, we get the following relationships:

$$(1)\quad \frac{\triangle BQC}{\triangle BQA}=\frac{CF}{FA}=t,\quad \frac{\triangle BQA}{\triangle AQC}=\frac{BE}{EC}=\frac{\triangle ARC}{\triangle BRC}=\frac{AD}{DB}=r$$

$$\frac{\triangle BRC}{\triangle ABR}=\frac{CF}{FA}=t,\quad \frac{\triangle BPA}{\triangle CPA}=\frac{BE}{EC}=s,\quad \frac{\triangle CPA}{\triangle BPC}=\frac{AD}{DB}=r$$

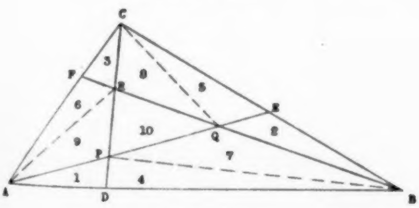

FIG. 14

where r, s, t are the ratios of the segments of sides AB, BC, CA respectively determined by taking D, E, F as arbitrary points of division. From Figure 14 we see that

$$(2)\quad \triangle ABC=\triangle BQC+\triangle BQA+\triangle AQC.$$

$$(3)\quad \triangle ABC-\triangle PQR=\triangle BRC+\triangle BQA+\triangle APC.$$

Hence

$$(4)\quad \triangle PQR=\triangle ABC-\triangle BRC-\triangle BQA-\triangle APC.$$

$$(5)\quad \triangle PQR=-\triangle BRC+\triangle BQC+\triangle AQC-\triangle APC$$

$$(6)\quad \frac{\triangle PQR}{\triangle ABC}=\frac{-\triangle BRC+\triangle BQC+\triangle AQC-\triangle APC}{\triangle BQC+\triangle BQA+\triangle AQC}$$

$$(7)\quad =\frac{\triangle BQA(t+1/s)-\triangle BRC-\triangle APC}{\triangle BQA(1+t+1/s)}$$

but

$$(8)\quad \triangle ABC=\triangle CPA+\triangle ABP+\triangle PBC$$

from which we get:

$$(9)\quad \triangle PQR=\triangle APB+\triangle PBC-\triangle BRC-\triangle BQA.$$

Equating (5) and (9) and substituting from (1) for r, s, and t, we get:

$$(10)\quad \triangle CPA=\frac{\triangle BQA(1+t+1/s)}{(1+s+1/r)}.$$

Also

$$(11)\quad \triangle ABC=\triangle BRC+\triangle CRA+\triangle ARB,$$

from which

$$(12)\quad \triangle PQR=\triangle CRA+\triangle ARB-\triangle BQA-\triangle APC.$$

Equate (9) and (12) and substitute from (1) as before to get:

$$\triangle BRC=\frac{\triangle BQA(1+t+1/s)}{(1+r+1/t)}.$$

Substituting the values obtained for $\triangle CPA$ and $\triangle BRC$ into (7) and simplifying we finally get:

$$(13)\quad \frac{\triangle PQR}{\triangle ABC}=\frac{(1-rst^2)}{(1+s+st)(1+r+rt)(1+r+rs)}.$$

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The second letter reads in part as follows:

In connection with the discussion of *nedians* which appeared in your Department of THE MATHEMATICS TEACHER I thought you might be interested in a plane geometry proof for the ratio

of the cevic triangle (and the menelaic triangle) to the given triangle. The only theorem used is that two triangles with equal altitudes are to each other as their bases.

Given (Figure 14):

$$\frac{AD}{AB}=\frac{BE}{BC}=\frac{CF}{CA}=\frac{1}{n}.$$

Prove:[1]

$$\frac{\triangle ABC}{\triangle PQR}=\frac{\triangle ABC}{\triangle 10}=\frac{n^2-n+1}{(n-2)^2}$$

$$\triangle ACD=\triangle ABE=\triangle BCF=\frac{1}{n}\cdot\triangle ABC,$$

since $\triangle ACD$ and $\triangle ABC$ have equal altitudes, therefore they are to each other as their bases AD and AB.

$$\triangle ACD+\triangle ABE+\triangle BCF=\frac{3}{n}\cdot\triangle ABC$$

$$\triangle ABE+\triangle BCF+\triangle ACD-\triangle 1-\triangle 2-\triangle 3+\triangle 10=\triangle ABC$$

$$(1)\quad \therefore\quad \triangle 1+\triangle 2+\triangle 3=\frac{3}{n}\cdot\triangle ABC-\triangle ABC+\triangle 10=\frac{3-n}{n}ABC+\triangle 10$$

$$\triangle 4=(n-1)\cdot\triangle 1,\quad \triangle 5=(n-1)\cdot\triangle 2,\quad \triangle 6=(n-1)\cdot\triangle 3$$

$$(2)\quad \triangle 4+\triangle 5+\triangle 6=(n-1)(\triangle 1+\triangle 2+\triangle 3).$$

$$(3)\quad \triangle 1+\triangle 2+\triangle 3+\triangle 4+\triangle 5+\triangle 6=n(\triangle 1+\triangle 2+\triangle 3).$$

Multiplying (1) through by n and using (3), we have

$$(4)\quad \triangle 1+\triangle 2+\triangle 3+\triangle 4+\triangle 5+\triangle 6=(3-n)\cdot\triangle ABC+n\cdot\triangle 10\cdots$$

but

$$\frac{\triangle 7+\triangle 10}{\triangle 9}=\frac{n-1}{1}$$

since the numerator and the denominator represent triangles with the same base, and which hence have the same ratio as their altitudes.

$$(n-1)\cdot\triangle 9=\triangle 7+\triangle 10$$

and similarly

$$(n-1)\cdot\triangle 7=\triangle 8+\triangle 10,$$
$$(n-1)\cdot\triangle 8=\triangle 9+\triangle 10$$

$$\therefore\quad (n-1)(\triangle 7+\triangle 8+\triangle 9)=\triangle 7+\triangle 8+\triangle 9+3\cdot\triangle 10$$

[1] *Department Editor's Note:* This result is a specialization of the one cited earlier, in which $r=s=t$, and with the proviso that $n=(1+r)/r$ since $AD/AB=1/n$ and $AD/DB=r$.

or

$$(5)\quad \triangle 7+\triangle 8+\triangle 9=\frac{3}{n-2}\cdot\triangle 10.$$

Adding $\triangle 10$ to both sides of the sum of (4) and (5) gives

$$\triangle ABC=(3-n)\cdot\triangle ABC+n\cdot\triangle 10+\frac{3}{n-2}\cdot\triangle 10+\triangle 10$$

which reduces to

$$\frac{\triangle ABC}{\triangle 10}=\frac{n^2-n+1}{(n-2)^2}.$$

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Central High School
Philadelphia 41, Penn.

55. Mathematics on the Move

Never has the volume of mathematical research and publication been greater than it is today. Never has the demand from industry and science for mathematicians, computers, and their products been greater. Although most of this work requires persons with graduate level training there are some places for persons with an undergraduate major in mathematics.

Although the demand, numerically, is not so great as to justify selling mathematics as a vocation to large numbers of secondary school or junior college students never-the-less more than a cursory knowledge of these situations should be the property of every teacher in order that he may (1) keep the guidance and administrative officers of his school aware of these needs and their implications for the curriculum; (2) locate, guide, and stimulate those students who might find pleasure and profit in continued study in the field; (3) give all of his students a better appreciation of the role and importance of mathematics today.

One of the best sources for data of this sort is "Professional Opportunities in Mathematics, A Report for Undergraduate Students of Mathematics" which appeared in *The American Mathematical Monthly,* volume 58, number 1, January 1951, pages 1–24 and which is available in reprint form from Professor M. H.

Gehman, Mathematical Association of America, University of Buffalo, Buffalo 14, New York, at 25 cents for single copies and 10 cents each for orders of 10 or more.

A further interesting aspect of this demand for mathematics is the recognition that "pure," "theoretical," research must also be encouraged if mathematics and mathematicians are to be available when needed. This too should be recognized by secondary and junior college teachers and administrators when planning curricula, teaching courses, motivating students, acting as guidance officers. (This does *not* justify teaching unmotivated, too abstract, too manipulative mathematics without pointing out and using its applications and connections with other fields, but rather makes it more important than ever that mathematics teachers "set their own houses in order" as to methods, curricula, course content, texts.)

Some interesting evidence for both the demand for mathematics and the recognition of the need for continued progress in pure mathematics is to be found in a recent letter from Marston Morse, Chairman of the Division of Mathematics of the National Research Council to the members of the American Mathematical Society.

Professor Morse called the attention of the members to foundations and offices which will offer financial support to research in mathematics in 1952–53. His partial list is abbreviated as follows:

1. *The Office of Naval Research and the Flight Research Laboratory* expect to make funds available again in the form of a few small contracts for support of individual research in theoretical mathematics, primarily post-doctoral research.

2. *The Office of Ordinance Research* has the support of basic research in mathematics among its functions.

3. *The National Research Council* again has funds from the Rockefeller Foundation for post-doctoral fellowships in the natural sciences, including mathematics.

4. *The National Science Foundation* expects to establish a fellowship program in the sciences, including mathematics, for the first, second, or third years of graduate study.

5. *Fulbright Awards* are available for university lecturing and post-doctoral research in all academic fields in many foreign countries.

Other recent developments creating demands for our products are the *National Mathematics Laboratory*, the growth of *operations analysis*, the tremendous strides in *automatic computation* and in the use of mathematics in various phases of the *social sciences*.

Can you tell your students about each of these? Can you cite them to places where they can read about them? Would you like more data on them? Perhaps you can send us references or your own "write up" explaining about one of them, or an outline of the project or exhibit which you or your students worked up about one of them?

The N.C.T.M.—M.A.A. Symposium at the University of Wisconsin

The First Symposium on Teacher Education in Mathematics will be held on the campus of the University of Wisconsin at Madison, August 26–30, 1952. It is being sponsored jointly by **The National Council of Teachers of Mathematics** and **The Mathematical Association of America.** Anyone interested in receiving further announcements of this symposium is invited to write to the Director, Rudolph E. Langer, 822 Miami Pass, Madison 5, Wisconsin.

INSTITUTES, MEETINGS, SUMMER SESSIONS

Boston University announces a summer **travel trip** for the **summer of 1953** for secondary school teachers of mathematics and science. A ten-week trip to countries in Western Europe will include meetings with leaders in education, visits to schools, museum exhibits in mathematics and science, and scientific laboratories. Ample time will be allowed for recreation as well. Complete details and itinerary will be available after August 1, 1952, by writing to Professor Henry W. Syer, Boston University School of Education, 332 Bay State Road, Boston 15, Mass.

The Second Annual Conference for Teachers of Mathematics will be held from **June 30 to July 12** on the **Los Angeles** campus of the **University of California**, according to Professor Clifford Bell, Head of the Mathematics Extension for the University. The purpose of this conference is to bring together teachers interested in mathematics—arithmetic through calculus—to study problems in the teaching of mathematics and to learn new uses of mathematics in various fields of endeavor. All who are interested in enriching the work being done in mathematics and in obtaining a broader concept of the place of mathematics in our present educational program are invited to attend and participate in the activities of the conference.

The conference will consist of two general session lectures and six study groups daily. In addition, a mathematics laboratory course will be offered where teachers may have the opportunity of studying and making various models and other aids in the teaching of mathematics. Two units of college credit may be obtained by participating in the conference and an additional unit may be earned by enrolling in the laboratory course.

The University of California's Department of Mathematics, Education and Mathematics Extension and University Extension are sponsoring the conference in cooperation with the California Mathematics Council and the National Council of Teachers of Mathematics. W. W. Rankin, Director and Professor of Mathematics, Duke University, will come to Los Angeles to preside at the conference. Additional members of the Planning Committee are: L. J. Adams, Chairman, Department of Mathematics, Santa Monica City College; Dale Carpenter, Academic Education Branch, Curriculum Division of the Los Angeles City Schools; Reuben Palm, Director, Secondary Education, Los Angeles County Schools; and Paul White, Head of the Department of Mathematics at the University of Southern California. A complete program may be obtained by writing to Clifford Bell, Mathematics Department, University of California, Los Angeles 24, California.

The Fourth Annual Conference on Teaching Mathematics, Grades 1–12, will be held at the **University of Wisconsin, July 21–25.** The program will consist of lectures, study groups and mathematics laboratories. An interesting recreational program has been planned. Housing will be available in student housing units. A conference for science teachers is to be held on the campus during the same week, and the two groups will jointly sponsor several sessions. For additional information, please write to Professor J. R. Mayor, North Hall, Madison 6, Wisconsin.

New York University announces the following courses in mathematics education for the **summer session June 30–July 11:** Applications of Mathematics and Teaching Aids in Mathematics Education, by Dr. Irving Dodes; The Teaching of Junior High School Mathematics, by Mr. Harry Ruderman; The Teaching of Arithmetic, by Miss Irene Harrison; Current Trends in Mathematics Curricula and Teaching, by Dr. John J. Kinsella; Testing and Evaluation in Mathematics Education, by Dr. Kinsella.

A workshop in Curriculum and the Teaching of Mathematics will be conducted by Dr. Kenneth E. Brown, Specialist in Mathematics of the U. S. Office of Education at the **University of Colorado** at Boulder from **July 24 to August 26.**

Duke University's Twelfth Annual Institute for Teachers of Mathematics will be held from **August 5 through August 15.** Professor W. W. Rankin, Director, has announced the general theme for this year's Institute as "Mathematics at Work." More than 1100 teachers from 37 states and a number from Canada have attended the Institute in the past 11 years. The purpose of the Institute is to bring together high school and college teachers of mathematics to study intensively problems of common interest and to learn new uses of mathematics in industry, business, science, and engineering, and to get acquainted with current teaching aids and the present curriculum trends in mathematics. Free and open discussions will be a feature of the Institute. Certificates of attendance will be issued upon request.

The Mathematics Laboratory will be open during the entire period of the Institute. This laboratory makes available in one place a wide range of materials relating mathematics to science, industry, engineering, education and commerce. The laboratory contains a large collection of present-day textbooks; a selection of books dealing with the applications of mathematics to science, engineering, industry and commerce; books on the history of mathematics, the philosophy of mathematics, mathematical recreations and puzzles; curriculum studies, tests, charts, graphs, and models. Recent additions are a wind tunnel model airplane, a model battleship (North Carolina), a number of mathematical computing instruments, a chronometer, replicas

(*Continued on page 390*)

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by JOHN R. MAYOR *and* JOHN A. BROWN
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IN THE November, February and April numbers of THE MATHEMATICS TEACHER for the 1950–51 school year three sets of questions on Mathematics Enrollments, General Mathematics, and Third Year Mathematics, respectively, were published in this Department of THE MATHEMATICS TEACHER. All three sets of questions were given again in the November, 1951 number. Preliminary reports on answers received have been published in the April, 1951 and the November, 1951 numbers.

By the date of final tabulation, February 10, 1952, replies to one, two or all of the sets of questions had been received from 121 schools in 82 of which a total of 79,189 students were enrolled. The enrollments in the other 39 schools were not reported. The replies came from 31 states and Canada. Six of the schools are private schools and four are parochial schools. Wisconsin schools contributed 20 of the replies; 9 came from Minnesota; 8 from Indiana; 7 each came from Iowa, New York, and Pennsylvania; and 6 each came from Illinois, Michigan, and Ohio. Other states from which 3 or more replies were received are California, Florida, Massachusetts, Oregon and Texas.

A tabulation of grade organization from which replies were received is as follows: 9–12, 46; 7–12, 25; 10–12, 19; other, 6; not given, 25.

Early replies reported in April, 1951, to questions on Mathematics Enrollments probably were based on the 1949–50 school year. About half of the remaining were based on the 1950–51 school year and half on the 1951–52 school year.

For the reports from schools with grades 9–12, slightly more than half came from schools of enrollment over 500, and six of these had an enrollment of over 2,000. Fifty-eight per cent of the replies from schools with grades 7–12, came from schools of enrollment over 500. There were no answers from schools of enrollment smaller than 100 and there were only 6 replies from schools of enrollment less than 200. Hence the answers are more representative of the larger schools.

Eighty-nine different schools reported on the questions on Mathematics Enrollments, 92 schools reported on practices in General Mathematics, and 89 schools on Third Year Mathematics.

The special nature of schools reporting in this study is probably best revealed by the answers to question 5 on Mathematics Enrollments, "What per cent of your graduates of last spring entered college this fall?" In answer to this question, 6 schools reported 100%, while 14 additional schools reported that more than 60% of their graduates entered college. Forty-one per cent of the schools, for which answers to this question were given, reported that more than 40% of their graduates entered college.

A. *Mathematics Enrollments*

Answers to the question on the requirements in mathematics for graduation were:

Number of Years Required	*Number of Schools*
0	12
1	43
$1\frac{1}{2}$	2
2	20
$2\frac{1}{2}$	1
3	9
4	1
Proficiency Test	1
Total	89

It is of special interest to note that for 21 of these schools it was indicated that the requirement had been in effect more than 20 years and for 8 of the 21, more than 40 years. Nearly 70% of the schools that answered this question indicated that the requirement had been in effect at least 10 years. It should be safe to assume that the requirement for the 33 schools for which no answer was given to this question had been in effect for a reasonably long time, else the question would have been readily answered.

The reports on per cents of graduates of the year on which the report was based who had completed one, two, three or four years of mathematics in grades 9–12 show that in 70 of the schools (more than three-fourths of the schools) 100% of the graduates had had one year of mathematics and that in all but one of the schools reporting more than 60% of the graduates completed one year of mathematics. Twenty-seven schools reported that 100% of their graduates had completed two years of mathematics; 63 of the schools reported that more than 60% of their graduates completed two years of mathematics (a requirement in only 36% of the schools); while all but one of the schools reported that more than 20% of their graduates completed two years of mathematics.

Only 12 schools reported that as many as 60% of their graduates completed three years of mathematics. The reports from more than half of the schools show that more than 30% of their graduates completed three years of mathematics, while in three-fourths of the schools more than 20% of the graduates completed three years of mathematics. Seventy-three per cent of the schools reported that less than 20% of their graduates completed four years of mathematics. It is interesting to observe in the replies that the per cents of graduates completing three years of mathematics and the per cents of the graduates who entered college rather closely parallel each other.

B. *General Mathematics*

Eighty of the schools, or 87%, reporting in this part of the study indicate that both algebra and general mathematics (or a course which might be called general mathematics) are offered. In 71 of the schools, or 77%, both algebra and general mathematics are offered in the ninth grade. Considerable variation is shown in the per cents of the students who take general mathematics as their first mathematics course in grades 9–12. The answers to this question were:

% of students taking general mathematics as first mathematics course in grades 9–12.	100	80–99	60–79	40–59	20–39	0–19	Not Stated
No. of schools reporting	9	1	9	31	15	12	15

Question 3 on General Mathematics was intended to determine what data are used in placement of students in General Mathematics. The number of schools reporting use of the following kinds of data was as follows:

Algebra aptitude test	27
Arithmetic achievement test	33
Grade in eighth grade mathematics	53
Intelligence test score	39
Written recommendation of teacher	39
Other	41

While many of the schools reported use of combinations of two or more of the kinds of data listed, no pattern of combination stood out predominantly in the replies. The most frequently mentioned combination which was reported by 9 schools, was the use of an arithmetic achievement test, grade in eighth grade mathematics, intelligence score, and written recommendations of teachers.

In reply to the question on what course students who have completed one year of general mathematics may elect, 69 of the schools indicated algebra, 24 marked a second year of general mathematics, 15 marked plane geometry and 28 reported

other courses in mathematics, with many schools marking more than one of the possible replies.

The question on the acceptance by colleges of general mathematics on the same basis as algebra was somewhat ambiguous and consequently difficult to answer. A large number of schools indicated that the answers varied for colleges which their graduates were attending, while 15 did indicate that colleges were accepting general mathematics on the same basis as algebra.

C. Third Year Mathematics

The replies to the first question of this part of the study can be seen by repetition of the question with the number of schools marking the various choices listed at the right:

What courses do you offer for the third year of sequential mathematics?

Third semester of algebra and a semester of solid geometry	20
Third semester of algebra and a semester of trigonometry	21
Second year of algebra	42
Year of plane geometry	26
Algebra and a subject in combination different from the above	6
Other	12

Again a number of schools marked more than one choice. The schools for which plane geometry was marked probably include both schools which offer general mathematics as the first course in the sequence and schools which offer two years of algebra before any plane geometry.

Only 11 schools reported ability grouping in third year sequential mathematics.

Nineteen of the 89 schools reporting have mathematics clubs and in nine of these the club is open to juniors and seniors. In three schools the clubs are open to sophomores, juniors, and seniors and in three the club is open to all interested. Two schools have clubs for seniors only and one has a club for juniors only and one for freshmen only.

Almost twice as many schools reported not using films in their third year course as reported using them. The film on the slide rule was listed by 11 schools and several reported using films on locus and on congruent triangles. These latter schools of course were those teaching plane geometry in the third year.

Forty schools reported that their students use the library as part of the suggested study in their third year course and 45 answered "no" on this question.

General Conclusions

The editors of this Department recognize that this study can not claim to be representative of practices in our schools and that the results must be used with caution. Nevertheless it is hoped that schools may find these summaries of some use in developing their own mathematics programs and that those seeking clues to current trends may find some answers here, in trends which seem rather strongly suggested. It is hoped that in another year through this Department, or through some other agency of the National Council, a study of this kind can be carried out for which a larger number of replies will be obtained and/or a more scientific sampling used.

The study does seem to indicate that present mathematics requirements for graduation have, in general, been in effect for more than ten years and that the most commonly occurring requirement is one year of mathematics. The study suggests that although it is not necessarily a requirement that in the majority of schools of enrollment over 100 more than half of the graduates complete two years of mathematics. The per cent of graduates who complete four years of mathematics apparently remains small, indeed considerably smaller than present and future needs for scientists and engineers demand.

It probably can be safely concluded from this study that a large part of the larger schools, at least, offer both algebra and general mathematics and that in most instances these courses are offered to ninth graders. There appears to be no definite pattern for placement of students

in general mathematics and this suggests a growing need for study of this problem, perhaps under sponsorship of the National Council. The fact that a variety of courses are open to students who complete one year of general mathematics seems to indicate that this course may not be planned, in many instances, as a terminal courses and suggests that there are marked differences among our schools in the objectives of such a course, with some certainly considering it a preparatory course for algebra. It is encouraging to observe that more than one-fourth of the schools in this limited sample offer a second year of general mathematics.

The variation in courses offered for the third year of sequential mathematics appears from the study to be greater than the editors of this Department had supposed. It is strongly suggested that only a small part even of our large schools use ability grouping in the advanced courses.

The small number of these schools using films should not be taken as indication that films are not being used very frequently since there are fewer good films for use at this level probably than for any other year. As a matter of fact, many teachers not using films expressed a desire to use films in third year mathematics if good ones could be suggested to them. While the replies on the use of the library give a fairly satisfactory picture it must be recognized that these schools are probably not typical, and that more teachers certainly need to be encouraged to use the library as part of their classwork and to organize mathematics clubs. It would be interesting to have a follow-up on these points in five years to determine if the many fine materials now available in The Mathematics Teacher serve to encourage these commendable practices.

Career Panel Discusses Engineering

Through the efforts of John K. Hefferman, Head of the Mathematics Department of Belleville High School, Belleville, New Jersey, a panel discussion for students interested in engineering was held in February in the high school auditorium. Parents of these students were also invited to attend. The superintendent of schools acted as moderator of the panel which consisted of four experienced engineers, representing the fields of chemical, civil, mechanical, and electrical engineering. Opportunity was given to parents and students to ask questions.

In preparation for this program, engineering material had been placed in the school library. All interested students submitted outlines of a term paper on engineering, and oral reports were given in classes. Students also saw movies in relation to their chosen career. Integration with the English department on the planning and writing of the term reports progressed smoothly.

It was emphasized by the speakers that never in American history have the opportunities for engineers been more challenging than they are today. Engineers are desperately needed for an unparalleled expansion of our peacetime industries, continued developments of our national resources, and the accelerating program of mobilization for national defense. The Help Wanted advertisements in any newspaper tell this story of manpower shortage in the engineering profession.

At the close of the meeting a suggested information sheet was given to all students and parents present. Mr. Carey H. Brown, Chairman, Engineering Manpower Commission of Engineers Joint Council, who prepared the information on the sheet, suggested that there are many other openings in fields closely allied to engineering. Corresponding to every engineer there is a need for additional persons who serve as mechanics, technicians, and technologists of one sort or another and who possess some of the interests and abilities of the engineer but by no means all of them. Here, too, for years to come, opportunities will abound for young men

and women who can do things with their hands and who crave the chance to share in the adventure of making the world a better place to live in the beginning of this atomic age.

Reported by HERMAN D. KNUPPEL
Office of Supervisory Staff of the Public Schools
Belleville, New Jersey

EVANSVILLE TEACHER INVENTS GRADOSCOPE

Teachers in Evansville, Indiana, have been helping one of their colleagues give a rigorous try-out to one of his several interesting inventions. Leroy W. Shrode, mathematics teacher at Central High School, has invented a piece of equipment which he has named the "Gradoscope." It is a device for simplifying and speeding up the process of marking papers. All teachers know what a burden the marking of papers is, and how impossible it is to mark every paper handed in. As a result, many papers do not get graded at all, and tests are postponed as long as possible, knowing that tests *must* be graded.

The device is, in effect, a piece of furniture which sits on top of the teacher's desk during the marking process. It is a plane, slightly inclined from the vertical, with sets of pegs which hold specially designed strips of paper. A correct set of answers is hung at the extreme left, and the answers are graded ***horizontally*** by stroking through errors with a soft colored pencil. Many teachers realize that to be really objective, Question 1 should be graded on all papers of a set before Question 2 is looked at. To do this with papers arranged in the traditional manner makes the job tremendous. With the Gradoscope, that manner of grading is the logical procedure because of the convenience of the arrangement. Errors have a way of standing out and calling attention to themselves (like a pupil raising his hand) when all answers to Question 1, 2, or 25 are on a single horizontal line.

When the marking of errors is finished, the assigning of a per cent grade is made very easy by an ingenious grading scale at the bottom of each strip. At that place there are three columns: Wrong, Right, Per cent. Ordinarily, the marker counts the wrongs and marks that number on the grading scale. If nearly all answers on a paper are wrong, the marker may, if he chooses, count the rights and mark that number. The per cent grade is automatically indicated.

Mr. Shrode and his friends admit that "short answer" tests have their limitations, and are not a universal substitute for tests in which the pupil must express himself fully. But when one considers the physical impossibility of grading very many tests during one report card period from all his classes, it becomes evident that some time-saver must be adopted. The Gradoscope is designed to be one of those time-savers. It is equally suitable for true-false, multiple choice, and direct answers, especially arithmetic and algebra.

When the teacher has finished grading a set of papers for a class, it is the experience of those who have tried it that they are not exhausted and disgusted, as they so often are when they have finished grading a set in the traditional manner. Timed experiments have been made which show that the time needed for grading a set is cut down to $\frac{1}{3}$ on answers which are hard to grade horizontally, and to less than $\frac{1}{4}$ on answers easy to grade thus.

The answer strips which Mr. Shrode has devised are about $1\frac{1}{4}$ inches wide, and about 14 inches long. An ingenious system of alternately colored areas very effectively guides the eye across the set of papers hanging on the Gradoscope. There are forms for 25, 40, and 50 answers. The form for 25 answers may be used just as easily for 20, as it has a double grading scale. The stock used is heavy ledger paper of good surface. Its weight eliminates the flimsiness experienced when regular theme or test paper is cut into narrow strips. Holes punched at the top and bottom permit hooking each strip onto properly

spaced pegs at the top and bottom of the Gradoscope. The Gradoscope itself is made principally of aluminum, with some stainless steel fittings. It is beautifully tooled and expertly made in every way. The aluminum parts are anodyzed a pleasing dark green by a procedure perfected by Mr. Shrode himself, who has studied even farther in advanced chemistry and physics than in mathematics. Both the Gradoscope and the answer strips have gone through numerous experimental stages, and both are amply covered by patents.

One more very great advantage of the Gradoscope must be mentioned. After a set of papers has been graded, before the strips are removed from the supporting hooks, the teacher immediately has a perspective of the general outcome of the test. In vertical columns, the papers of pupils who missed nearly everything stand out prominently, as do the 100's and nearly 100's. In horizontal rows, the questions which were almost unanimously missed stand out and call attention to themselves, as do also the questions gotten right by nearly everybody. Then, after making such a "rough and ready" appraisal, the teacher can easily make a count of the "number of times missed" for every question in the test by simply hanging a clean answer strip at the extreme right and recording his count on it, and he can even make notes of the *manner* in which certain typical or especially annoying mistakes occurred, to be used in reteaching the unit. This quick appraisal and statistical summarizing is perhaps as valuable a feature of the Gradoscope as the ease of marking itself.

Henry Meyer
Central High School
Evansville, Indiana

Institutes, Meetings, Summer Sessions

(*Continued from page 384*)

of seventeenth-century clocks, an anti-aircraft firing detector, a Cadillac engine and hydramatic drive, and a jumbo gyroscope.

Principal speakers and their topics include: William H. Ruffin, President, Erwin Mills, Immediate past president of National Association of Manufacturers, "Industry Takes a Look at Education"; D. G. Sharp, Professor of Biophysics, Medical School of Duke University, "Mathematics in Virus Research"; John W. Cell, North Carolina State College, "Unusual Graphs for Well-Known Functions"; Louis P. Allen, Staff Oceanographer, Naval Research, Washington, D. C., "Figuring the Weather"; Admiral Alfred C. Richmond, United States Coast Guard, Washington, D. C., "Loran—Navigation without Mathematics?"; Phillip Jones, University of Michigan, "Techniques for Teaching for Meaning"; A. E. Roach, Mechanical Engineer, General Motors Research Laboratories, Detroit, Michigan, "Similitude—A Fusion of Mathematics and Experimentation"; W. A. Gager, University of Florida, "Between Proper Limits"; Harold W. Lewis, Professor of Physics, Duke University, "Nuclear Research Program at Duke University"; W. M. Whyburn, Chairman of the Department of Mathematics, University of North Carolina, "Mathematics for the Muddled."

Friday, August 8 has been set aside as "School Administrators Day" and a special program for superintendents, principals, supervisors and heads of departments has been arranged. In addition ten study groups have been planned which will be led by Allene Archer, Richmond, Va.; Ida May Bernhard, San Marcos, Tex.; Amelia Richardson, McKeesport, Pa.; L. G. Woodby, University of Michigan; Veryl Schult, Washington, D. C.; Ella Marth, Washington, D. C.; Frances Burns, Oneida, N. Y.; Nanette R. Blackiston, Baltimore, Md.; Frances Johnson, Oneonta, N. Y.; Ruth Fanning, Asheville, N. C.; and Henry A. Shannon, North Carolina State Department of Mathematics.

Copies of the program may be obtained from Professor W. W. Rankin, Duke University, Durham, N. C.

Massachusetts Institute of Technology announces two special courses for scientists, economists and engineers which will be given during the **1952 Summer Session**. A course in Theory of Games will be given from June 9 to July 18 by Professor A. W. Tucker of Princeton University and a course in Special Functions from July 21 to August 29 by Professor Hans Rademacher of the University of Pennsylvania.

The **National Conference on Safety Education in Elementary Schools** will be held on the campus of **Indiana University, August 18-22**, under the direction of NEA's National Commission on Safety Education. The purpose of this working conference will be to give educators the opportunity and all possible help in arriving at good practices and methods of safeguarding our

(*Continued on page 395*)

Annual N.C.T.M. Summer Meeting with N.E.A.

Detroit, Michigan, Monday, June 30, 1952

9:00 A.M. **Registration**—Veterans' Building—Polar Bear Room

9:30–11:45 A.M. **First Session**—Veterans' Building—Polar Bear Room

Theme: *Some Proposals for Progress*

Presiding: C. H. Butler, Western Michigan College of Education, Kalamazoo, Michigan

The Superior Student—How To Locate Him and What To Do with Him

E. H. C. HILDEBRANDT, Northwestern University, Evanston, Illinois.

Programs for General Education and Functional Competence—High School and College

H. VERNON PRICE, State University of Iowa, Iowa City, Iowa.

A Program for Competent Students Lacking Technical Interests or Objectives

ELMER McDAID, Detroit Public Schools

Discussion

12:15 P.M. **Luncheon**—Veterans' Building—Rainbow Room

(Luncheon, $2.45. Send reservations and check to R. A. Morley, 18701 St. Marys St., Detroit before June 24, 1952.)

1:30–2:45 P.M. **Second Session**—Veterans' Building—Polar Bear Room

Theme: *The Need for Mathematics in Business and Industry*

(This session has been planned with and manned through the cooperation of the Industrial Mathematics Society of Detroit.)

Presiding: C. C. RICHTMEYER, Central Michigan College of Education, Mt. Pleasant, Michigan.

The Emergence of Mathematics Needs in an Industrial Center

ARVID JACOBSON, Wayne University, Detroit, Michigan, Chairman, Education Committee, Industrial Mathematics Society

Mathematics in the Skilled Trades

CARL J. CARLTON, Head, Related Training Unit, Trade and Technical Section, Ford Motor Company Training Department

Mathematics in Engineering

ROBERT SCHILLING, Research Laboratories Division, General Motors Corp.

Mathematics in Retail Business

E. C. STEPHENSON, Vice-President, Finance and Accounts, J. L. Hudson Co.

2:45 P.M. **Intermission**

3:00 P.M. **Panel Discussion**—Veteran's Building—Polar Bear Room

Chairman: FRANKLIN FREY, Cass Technical High School, Detroit, Michigan

Participants:

Messrs. JACOBSON, CARLTON, SCHILLING, STEPHENSON, plus A. P. FUGILL, Systems and Stations Engineer, Detroit Edison Company

DR. W. H. HULSWIT, Manager, Tire Engineering Research, U. S. Rubber Co.

DR. C. V. WINDER, Research Pharmacologist, Parke, Davis and Company

General Chairman

AGNES HERBERT, Clifton Park Junior High School, Baltimore, Maryland

Local Program Chairmen

PHILLIP S. JONES, University of Michigan, Ann Arbor, Michigan

FRANKLIN FREY, Cass Technical High School, Detroit, Michigan

Local Arrangements

DETROIT MATHEMATICS CLUB, R. A. MORLEY, President

BOOK SECTION

Edited by JOSEPH STIPANOWICH
Western Illinois State College, Macomb, Illinois

BOOKS RECEIVED

High School

The New Applied Mathematics (Fourth ed.), by Sidney J. Lasley and Myrtle F. Mudd. Cloth, xii+387 pages, 1952. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N. Y. $2.56.

Mathematics, A Second Course, by Myron F. Rosskopf, Harold D. Aten, and William D. Reeve. Cloth, xviii+365 pages, 1952. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $2.80.

College

College Algebra, by Ross R. Middlemiss, Washington University. Cloth, xix+344 pages, 1952. McGraw-Hill Book Company, 330 West 42nd St., New York 18, N. Y. $3.50.

Elements of Statistics (Second ed.), by Elmer B. Mode, Boston University. Cloth, xvi+377 pages, 1951. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N. Y. $4.75.

Graphic Aids in Engineering Computation, by Randolph P. Hoelscher, University of Illinois; Joseph N. Arnold, Purdue University; and Stanley H. Pierce, University of Illinois. Cloth, viii+197 pages, 1952. McGraw-Hill Book Company, 330 W. 42nd St., New York 18, N. Y. $4.50.

The Philosophy of Mathematics, by Edward A Maziarz, St. Joseph's College. Cloth, viii+286 pages, 1950. Philosophical Library, Inc., 15 East 40th St., New York 16, N. Y. $4.00.

Contributions to the Founding of the Theory of Transfinite Numbers, by Georg Cantor. Paper, ix+211 pages. Dover Publications, 1780 Broadway, New York 19, N. Y. $1.25 (Cloth, $2.75).

A Philosophical Essay on Probabilities, by Pierre Simon, Marquis de Laplace, translated from the Sixth French Edition by Frederick Wilson Truscott and Frederick Lincoln Emory. Paper, viii+196 pages, 1951. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $1.25. (Cloth, $2.50.)

Calculus, by John F. Randolph, University of Rochester. Cloth, x+483 pages, 1952. Macmillan Co., 60 Fifth Ave., New York 11, N. Y. $5.00.

Life Insurance Case Analysis, Methods and Materials, by Henry T. Owen, University of Texas. Paper, vi+109 pages, 1952. Prentice-Hall, Inc., 70 Fifth Ave., New York 11, N. Y. $2.50.

Brief Trigonometry, (Revised ed.), by Edward A. Cameron, University of North Carolina. Cloth, vi+153 pages, 1952. Henry Holt and Co., 383 Madison Ave., New York 17, N. Y. $2.10.

The Elements of Mathematical Analysis (Second ed.), 2 Vols., by J. H. Mitchell, and M. H. Belz, University of Melbourne. Cloth, xxiii+xi+1087 pages, 1952. Macmillan Co., 60 Fifth Avenue, New York 11, N. Y. $13.60 set.

Nomography and Empirical Equations, by Lee H. Johnson, Tulane University. Cloth, ix+150 pages, 1952. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N. Y. $3.75.

Miscellaneous

Film and Education, by Godfrey M. Elliott. Cloth, xi+597 pages, 1948. The Philosophical Library, Inc., 15 East 40th St., New York 16, N. Y. $7.50.

The Nature of Number, by Roy Dubisch, Fresno State College. Cloth, xii+159 pages, 1952. Ronald Press Company, 15 East 26th St., New York 10, N. Y. $4.00.

The 1952 Blue Book of 16 mm. Films. Paper, 172 pages, 1952. The Educational Screen, Inc., 64 E. Lake Street, Chicago 1, Ill. $1.50.

REVIEWS

Everyday General Mathematics, Book II, William Betz, A. Brown Miller, F. Brooks Miller, Elizabeth Mitchell, and H. Carlisle Taylor. Boston, Ginn and Company, 1951. ix+438 pp., $2.60.

This book is the second of two volumes written for the second track course in high school mathematics, commonly known as general mathematics, and is a continuation of the work started in Book I.

The textbook material is very flexible and presents three major adaptations: (1) for groups that have had some algebra, (2) for groups that have had no algebra, and (3) for groups whose interests are definitely vocational.

Time schedules for the first two courses are suggested in the textbook. Cumulative reviews help measure the student's progress in mastering the mathematical essentials. There is a chapter test at the end of each chapter, and the objectives of each chapter are definitely outlined in the chapter summaries. The problems are well graded and provide motivation through practical applications to home life, community life, shop work, science, aviation, and statistics. The pictorial illustrations are excellent. The print is

large and clear and the material is well spaced.

The book contains a table of squares and square roots of numbers and a table of sines, cosines, and tangents. A teachers' manual with teaching suggestions is available.—WINNIE MACON, Haskell Institute, Lawrence, Kansas.

A First Course in Algebra (Second ed.), Walter W. Hart. Boston, D. C. Heath and Company, 1951. viii+389 pp., $2.28.

This elementary algebra has the virtues and the faults of great experience in the writing of books and of careful catering to the trade. The format is good, the explanations simple and clear, the exercises are complete and excellently graded. There is an abundance of good review exercises and of tests. Nearly every page is a unit in itself. The busy teacher can go through it page by page and be sure that a reasonably diligent and uninquisitive class will not be too much baffled. The explanations of minutiae are, on the whole, clear and good.

There is a serious slip up in the use of the word "cancel." On page 146 "canceling terms in an equation" is explained as an optional "short cut." In this connection "canceling" means adding or subtracting the same number to both sides of the equation. On page 229, in connection with the procedure for reducing a fraction to its lowest terms, there occurs the following: "Caution. You must divide N (numerator) and D (denominator) by a common factor of them. Moreover, remember you are dividing; do not think of it as "canceling'." On page 232, after giving the Rule: "The product of two or more fractions is the product of their numerators divided by the product of their denominators" the author, in the next sentence, says, "Divide out (or cancel) any factors which appear in both a numerator and a denominator."

The pupil whose teacher does not go beyond the level of this book will have little conception of the broad general ideas of algebra. He will have no inkling of the postulational basis of the subject, no conception of algebra as a device for facilitating quantitative thinking, no clear ideas of the importance and usefulness of type forms, no hint that graphing, by setting up a one-to-one correspondence between pairs of numbers and points in the plane ties algebra to geometry. In short, the book does little to increase the student's powers of generalization. Instead it will leave him with some skill in mechanics.—JACKSON B. ADKINS, Phillips Exeter Academy, Exeter, New Hampshire.

Trigonometry For Today, Milton Brooks, A. Clyde Schock with I. Oliver, Jr., Consultant. New York, Harper Brothers, 1951. ix+306 pp. including tables, $2.96.

Twenty years of classroom experience has enabled the authors to prepare this text based on materials collected during this time. The topics given consideration are (in their order of presentation): Functions and Graphs, The Trigonometric Functions, The Fundamental Identities, Graphs of the Trigonometric Functions, The Reduction Rule, Right Triangles, Functions of Two Angles, Oblique Triangles and Supplementary Topics.

Recommendations of groups such as the Pennsylvania State Committee were followed when preparing the general approach and order of the topics considered. The student should grasp the necessary concepts if he notes the following learning devices utilized by the authors. Important concepts are printed in either italicized or bold faced type. Important suggestions and rules are stated in bold faced type within a blocked off portion of various pages throughout the text. Many exercises are included so that the student may have an opportunity to check his acquisition of important concepts. Near the end of each chapter is another rectangular area containing an average of five questions dealing with important principles of the chapter. The last safeguard is in the form of summary tests at the close of each chapter.

Various discussions and exercises attempt to relate trigonometry to other fields such as geometry, algebra and the physical sciences. The last portion of this work is devoted to a collection of tables which the student will need as he progresses.—RODERICK C. MCLENNAN, Arlington Heights Township High School, Arlington Heights, Illinois.

Trigonometry, Cecil T. Holmes. New York, McGraw-Hill Book Company, 1951. ix+246 pp., $3.00.

This text is designed for college freshmen who have had no previous trigonometry. It stresses understanding of the basic concepts as well as preparation for analytic geometry and the calculus.

One of the many fine features of this book is the repeated use of analytic geometry methodology. By employing rectangular and polar coordinates and the distance formula the author handsomely derives cos(a-b), the law of sines, and the law of cosines. The use of directed line segments makes it very easy to represent the trigonometric functions by their line values and to introduce the student to the graphs of the trigonometric functions.

The topics treated are the conventional ones: coordinate systems, the trigonometric functions, radians, graphs, inverse functions, identities and conditional equations, common logarithms and logarithmic computation, and the solution of plane triangles. A chapter on spherical trigonometry and another on series, complex numbers and the number *e* are included for the more able student. These chapters may be omitted without destroying the continuity of the earlier material.

The chapters on logarithms and identities are expertly handled. The exercises for logarithmic computation are carefully selected from the areas of the physical and biological sciences and the field of commerce.

On pages 17 and 18 this reviewer found a

fine discussion of the meaning of tan 90°. On page 1, the author fails to emphasize the fact that a one-to-one correspondence between points on a number axis and real numbers is an assumption. The student might have the feeling that this relationship follows from the diagram and the written material which precedes it.

Bound in the book are five-place logarithmic and trigonometric tables and a table of the numerical values of the trigonometric functions. The definitions in the book are clear and the presentation of the subject matter is excellent. This book should be a welcome addition to the growing list of modern textbooks which stress functional competence and understanding in mathematics.—IRWIN K. FEINSTEIN, University of Illinois, Chicago Undergraduate Division, Chicago, Illinois.

Intermediate College Algebra, Edward M. J. Pease. New York, Prentice-Hall, Inc., 1950. vii+420 pp., $2.85.

The usual topics for a second course in algebra make up the contents of this book. In the preface the author states: "fundamentals of arithmetic and algebra are covered to such an extent that a good student may use this text for his first course in Algebra." This reviewer thinks that a first course which used and enlarged the understandings of arithmetic should precede the use of this book.

The author considers the extending of our number system to include all signed arithmetic numbers. This is followed by a chapter dealing with generalized number. Unfortunately, this puts a chapter of mechanical learning involving the following of rules and obeying little understood laws ahead of a chapter which permits of more intellectual activity on the part of the student.

Throughout the book the instruction is clear and definite. There is more emphasis on the "how" than on the "why" of procedures.

There are lists of questions which stimulate thought and direct emphasis. There are quizzes at the ends of the chapters. There are plenty of well-graded exercise lists. The word problems are excellent. There are very fine short discussions on topics such as Mathematics as a Language, Factoring, and others. The author has outstanding ability in making himself clear with few words.

There are answers to even numbered problems.

Any teacher of algebra will find interesting and helpful ideas in this book.—WILLIAM B. STORM, DeKalb, Illinois.

College Algebra, (Fifth ed.), Henry L. Rietz and Arthur R. Crathorne. Revised by J. William Peters. New York, Henry Holt and Company, 1951. xv+387 pages, $2.95.

This new edition of a well known text introduces some changes that have seemed desirable to the new co-author from his experiences with students. This edition includes a more extended review of topics of secondary school algebra, a first chapter on the number system and elementary operations that has been re-written, and revised exercises. The usual topics for college algebra are included. There are chapters on partial fractions, limits, and infinite series.

Notable features include emphasis on the number system and the early introduction and use of functional notation, graphs, determinants, and logarithms. Practical problems from many fields of study are given.

The wording of the text is rather limited and concise. The student is told how, but the "why" is not always evident. The student, if required to do individual study, might wish for more explanations on some of the topics.—FRANCIS R. BROWN, Illinois State Normal University, Normal, Illinois.

College Mathematics, A First Course (Second ed.). W. W. Elliott and Edward R. C. Miles. New York, Prentice-Hall, Inc., 1951. xii+436 pp. $4.85.

This text presents in one volume the essentials of college algebra and plane trigonometry and an introduction to plane analytic geometry and calculus. It is written especially for use in terminal courses in mathematics such as desired by students in the natural sciences or business administration. After the completion of such a terminal course the text will serve as a valuable reference for the student of science and business.

When used with students who expect to continue the study of mathematics beyond the first year, the instructor may select the topics that satisfy the prerequisites of the particular course that follows. There is ample material in the first three parts for a six-semester hour course or if more emphasis is to be placed on elementary topics, the book may well serve for a two-year course.

The arrangement of the material allows certain chapters to be omitted without interrupting the continuity of the text; this allows for individual differences in students, teachers and colleges.

Illustrative examples usually precede the general discussion but more examples and interpretation would be valuable. The text provides many opportunities for good teaching. There is an abundance of good exercises of all types and they provide a challenge to the student. Answers to the odd numbered exercises are included in the back of the book along with a set of mathematical tables, for the convenience of the users. Answers to all exercises are available in a separate booklet. A chapter on inequalities has been included on the recommendation of the many users of the first edition. An entire chapter is devoted to miscellaneous exercises.

The material covered is valuable as a basis for a terminal or first course in college mathematics and as a handy reference for teachers of high school or college mathematics.—GEORGE L. KEPPERS, Iowa State Teachers College, Cedar Falls, Iowa.

Nominations for the Board of Directors

The By-Laws of the National Council of Teachers of Mathematics require that "the Nominations and Elections Committee shall cause an announcement to be published in the official journal, at least five months before the annual meeting, inviting members of the Council to suggest nominees for elective offices."

The 1953 ballot will present candidates for the following offices:

A Vice-President representing junior high school mathematics (to serve for two years)

A Vice-President representing college mathematics (to serve for two years)

Three members of the Board of Directors (to serve for three years).

The candidates for Directors may represent any level of instruction. The present geographic plan requires that only one of the nine Directors may come from one state. Hence, the following states will not be represented on the ballot for the three Directors: Florida, Iowa, Kansas, Ohio, Texas, and Virginia. This limitation does not apply to officers.

The Nominating Committee invites members of the Council to suggest persons to be nominated and to submit brief statements of their qualifications and their participation in local and national affairs pertaining to mathematics. Suggestions should reach the committee as soon as possible, but not later than September 1, 1952. Suggestions may be sent to any member of the Nominating Committee: H. W. Charlesworth, East High School, Denver 6, Colorado; Agnes Herbert, Clifton Park Junior High School, Baltimore, Maryland; Lenore John, Laboratory School, University of Chicago, Chicago 37, Illinois; F. Lynwood Wren, Peabody College, Nashville, Tennessee; Edith Woolsey (Chairman), 5415 Aldrich Avenue South, Minneapolis 19, Minnesota.

Institutes, Meetings, Summer Sessions

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important human resources. All elementary school personnel interested should contact Norman Key, Secretary of the National Commission on Safety Education, 1201 Sixteenth Street, N.W., Washington 6, D. C.

The annual meeting of the **Louisiana-Mississippi Branch** of the **National Council of Teachers of Mathematics** was held at Northwestern State College, Natchitoches, Louisiana, February 15 and 16, 1952. This was the twenty-ninth joint meeting with the Louisiana-Mississippi Section of the Mathematical Association of America. Eighty-nine registered for the meeting of whom thirty-seven were Council members. Dr. W. M. Whyburn of the University of North Carolina was the guest speaker at the joint banquet held on Friday and spoke on "Efficient Use of Mathematically Trained Manpower." The program at the Council meeting included the following: "Practical Applications of Inequalities" by E. L. Clifford of Crowley, Louisiana High School; "Utilization of Research in Mathematical Methods" by Phillip W. Mouw of Southeastern Louisiana College Training School; "The Mathematics Teachers and Public Relations" by Elora Keyes of the Hattiesburg, Mississippi Junior High School; "The Use of Visual Aids in Teaching Geometry" by Mrs. Fred Cook of the Monroe, Louisiana High School; and "Spanning the Mathematical Gap Between High School and College" by E. H. Herron of Fair Park High School and Evening School of Centenary College. Miss Mary Emma Fancher of Hinds Junior College was elected Chairman for the 1952–53 academic year.

The Pennsylvania Council of Teachers of Mathematics held its **First Annual Meeting** at State Teachers College at Indiana, Pennsylvania on Saturday, March 29, 1952. Three sectional meetings were held in the morning. "Successful Teachers of Arithmetic," was the topic presented by Clara E. Cockersville, Assistant Superintendent of Schools, Armstrong County, at the Elementary School Section. At the Secondary School Section, Lee E. Boyer of Millersville State Teachers College spoke on "Enrichment Problems for the Algebra Class" and John Hoshauer of Edinboro State Teachers College discussed "Do's and Don'ts for Meaningful Teaching of Arithmetic." The two speakers at the College and Teacher Training Section were James S. Taylor, Head of the Department of Mathematics at the University of Pittsburgh who spoke on "The Importance of Presentation in Recreational Mathematics" and I. L. Stright of Indiana State Teachers College and Margaret Rhoads of Slippery Rock State Teachers College who discussed "Fundamentals of Mathematics—A required Course for All Prospective Teachers." The principal speaker at the afternoon meeting was Professor A. I. Oliver of the School of Education of the University of Pennsylvania whose topic was "Smiles of Mathematics." Dr. Catherine A. V. Lyons is President of the Pennsylvania Council.

Two brochures of interest to mathematics teachers have been made available by the Research Committee of the Pennsylvania Council. They are "Mathematics in the Planning of a Modern Watch" by George W. Sauerwald of the Hamilton Watch Company, Lancaster, Pennsylvania, and "Installment Buying Interest Formula" by Lee E. Boyer. Single copies may be secured at 25 cents each from Lee E. Boyer, Chairman, Research Committee of the PCTM, State Teachers College, Millersville, Pa.

NEWS NOTES

Newly elected officers of the **National Council of Teachers of Mathematics** were announced at the annual meeting at Des Moines, on April 17, as follows:

President, John R. Mayor, Madison, Wisconsin
Vice-President—Elementary School, Irene Sauble, Detroit, Michigan
Vice-President—Junior High School, Agnes Herbert, Baltimore, Maryland
Vice-President—Senior High School, Marie Wilcox, Indianapolis, Indiana
Board of Directors,
Allene B. Archer, Richmond, Virginia
Ida May Bernhard, San Marcos, Texas
Harold P. Fawcett, Columbus, Ohio

The First General Assembly of the **International Mathematical Union** was held in Rome, Italy, March 6–8, 1952. The Unione Matematica Italiana acted as host to the Assembly. Representatives from twenty-five foreign nations were in attendance. The five American delegates were: Marshall H. Stone, chairman of the department of mathematics at the University of Chicago, who was elected first president; Einar Hille, Yale University, past president of the American Mathematical Society; J. R. Klein of the University of Pennsylvania, former secretary of the American Mathematical Society; Saunders MacLane, University of Chicago, president of the Mathematical Association of America; and G. T. Whyburn, University of Virginia, president-elect of the American Mathematical Society.

The formation of an **Association for Multi-Sensory Aids for the Teaching of Mathematics** is announced from England. The Association will deal with the design, manufacture and distribution of teaching aids in mathematics. Individuals may help in one or more of several ways: 1) as a member of a small Central Committee, 2) as a local representative, 3) as an individual experimenting and bringing to perfection ideas suggested by the Central Committee, 4) as an individual producing detailed scripts, sketches of illustrations and teaching notes for films and filmstrips, 5) as an artist using appropriate tools and materials to produce high quality diagrams for development into filmstrips, and 6) as an individual wishing to be kept informed of the Aids produced. Types of aids to be manufactured include films, filmstrips, models, apparatus of various types, pictures, charts, illustrations, diagrams, and graphs. The first aid to be announced is a device for demonstrating that the area of a circle is equal approximately to a figure of area $=3\frac{1}{7}\ r^2$. Correspondence should be addressed to Mr. R. H. Collins, B.Sc., 47 Edward Avenue, Leicester, England.

Myron F. Rosskopf has been appointed Associate Professor of Mathematics at Teachers College, Columbia University effective July 1, 1952. Dr. Rosskopf has been in charge of the training of mathematics teachers at Syracuse University for the past four years.

The October 1951 issue of **Mathematical Pie** (see THE MATHEMATICS TEACHER, May 1951, page 364; also this issue page 373) is now out of print according to a report from the editor, Mr. R. H. Collins. The total edition of 35,600 copies was distributed to over 1000 subscribers and 50,000 copies of the February 1952 issue were made available.

A new set of mathematical films have been prepared by J. L. Nicolet, a Swiss teacher of mathematics. The entire series are listed under the name "Animated Geometry" and include the following titles: A Circle Is Determined by Three Points; Two Given Circles Seen At Equal Angles; Subtended Arc; Triangle Formed from Sides of Polygons; The Strophoid and the Golden Section; The Golden Section and the Regular Pentagon; Internal Bisectors of a Triangle; The Ratio Property of the External Bisector; A Given Line Seen at a Given Angle; The Angle at the Circumference; Construction of the Regular Pentagon; Locus of a Circle Tangent to Two Given Concentric Circles; Hypocycloid Motion with Circles in a Ratio of One to Two.

Nine new filmstrips on mathematics are listed in "The Gateway Series" under the following titles: The Quadratic Graph; From Experiment to Law; History of Calculation; An Area Construction; Graphical Solution of Equations; Permutations; An Introduction to Loci; How to Read Your Tables; An Introduction to the Calculus. Further information about both the films and filmstrips may be obtained from Mr. R. H. Collins, 47 Edward Avenue, Leicester, England.

The 31st Edition of **Automobile Facts and Figures** is now available. Copies are free and may be secured by writing to the Automobile Manufacturers Association, New Center Building, Detroit 2, Michigan.